Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 211 JANUARY, 1931 No. 1

TRANSIENT CURRENTS IN TRANSFORMERS.

BY

H. M. TURNER,

Associate Professor of Electrical Engineering, Yale University.

In developing the equation for the transient current in air-cored coils it is customary to express the induced voltage in terms of the rate of change of the current. When the magnetic circuit consists of a closed iron core, as in a transformer, the induced e.m.f. must be expressed by a more fundamental relation, namely, the rate of change of the flux. This is necessary for with iron the flux does not vary directly with the current but also depends upon the saturation of the core. Unfortunately the relation between the flux and current is, in general, unknown or cannot be expressed by any simple equation so a rigorous mathematical solution is not feasible. The method of finite differences [1, 2] may be used to obtain an approximate solution; if sufficient magnetization data are available. However, this method will not be illustrated here.

OBJECT.

The purpose of this paper is to present the fundamental principles underlying the phenomena of transient currents in transformers, when connected to a source of power, by starting with simple inductive circuits in the permanent state, which are well understood, and gradually leading up to

[1] T. D. Yensen, "Starting Currents of Transformers," Bulletin 55, University of Illinois.

[2] Dr. E. J. Berg, "Electrical Engineering," 1916, p. 62.

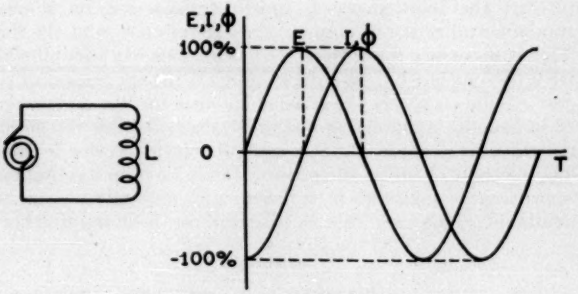
the more involved case of the transformer in the transient state. The elements affecting the transient current are considered individually. The manner in which the current depends upon the initial magnetic state of the core, the magnitude and phase of the impressed e.m.f. at the instant of closing the circuit, and various types of secondary loads are analyzed and experimentally verified. A sine wave of impressed e.m.f., $e = E \sin wt$, is assumed in all cases. In the interest of simplicity the discussion will be confined to single windings until loaded transformers are considered.

I. AIR-CORED COIL OF ZERO RESISTANCE.

(a) Permanent State.

In the normal permanent state the induced e.m.f. in an air-cored coil assumed to be without resistance is at all times equal to the impressed and requires for its generation a sine wave of flux and current lagging 90° behind the impressed voltage as shown in Fig. 1. To avoid the confusion of many

FIG. 1.



Permanent State for Coil Without Resistance.

curves the induced e.m.f. is not drawn but of course would be opposite in phase to the impressed. The point of closing the circuit is not indicated in this figure but to establish the normal permanent state, with the flux and current symmetrical with respect to the time axis, it would be at the maximum e.m.f. as will be evident later.

In the interest of generality the maximum values of E, I and Φ for the normal permanent state are taken as 100 per cent. Observe that when the e.m.f. is passing through zero and increasing in a positive direction the flux is $-$ 100 per cent. and 180° later is $+$ 100 per cent., that is, during the time the e.m.f. is positive the flux increases in a positive direction 200 per cent. During the next 180°, while the e.m.f. is negative, the flux decreases from $+$ 100 per cent. to $-$ 100 per cent. or a total change of 200 per cent. in the negative direction.

(b) Transient State.

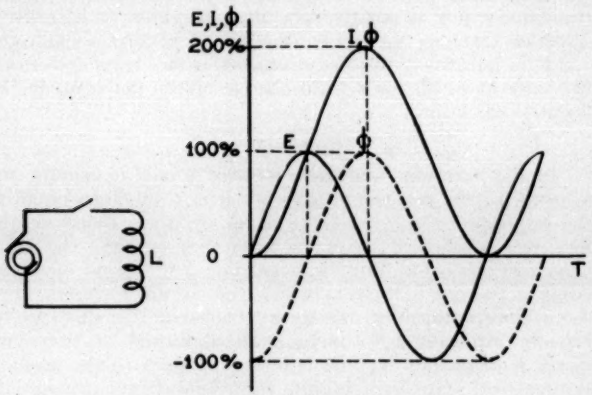
In the transient state the induced e.m.f. is exactly the same as for the permanent state for it is at all times equal to the impressed e.m.f., as there is no iR drop, therefore the flux must change at the same rate, that is, have the same wave form as before but its magnitude will depend upon the point of the e.m.f. wave at which the circuit is closed. For the sake of uniformity, the term "transient," in this case, is broadly interpreted to include all abnormal or non-symmetrical conditions even though the approach to the normal, symmetrical state is of infinite duration. Since a magnetic field represents stored energy it cannot be established in zero time, as this would require that work be done at an infinite rate which is obviously impossible, therefore the initial flux and current will be zero at the instant the circuit is connected regardless of the point of closing. Knowing these curves start from zero and that their rate of change or wave shape is the same as for the normal state their magnitude is completely determined.

Briefly, for a given air-cored coil without resistance, the impressed e.m.f. determines the induced e.m.f. hence the rate of change of flux and its wave shape, and the point on the impressed wave at which the circuit is closed determines the magnitude of the flux and therefore the current.

If the switch is closed when the e.m.f. is passing through zero increasing in a positive direction the flux and the current increase from 0 to $+$ 200 per cent. during the first 180° and decrease from $+$ 200 per cent. to 0 during the next 180° so they have the same values at the end of the first cycle as at

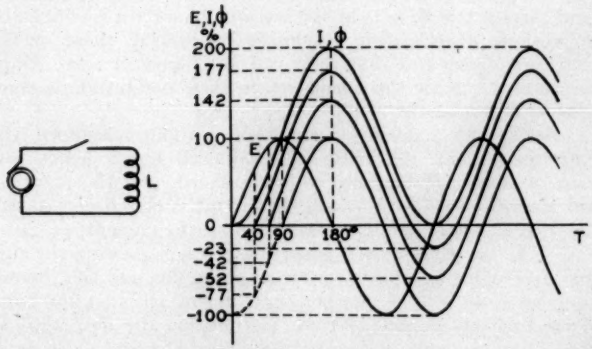
the beginning which means that all succeeding cycles will be similar to the first being permanently displaced upward 100 per cent. as indicated in Fig. 2.

FIG. 2.



Transient State Closing Circuit at Zero E.M.F.

FIG. 3.



Transient State Closing at Different Points.

The effect of closing the circuit at various points on the e.m.f. wave is illustrated in Fig. 3 which shows how the curves are displaced.

The following tabulation emphasizes important points to be noted.

TABLE I.

Close at wt.	Flux and Current.				
	Normal.	Initial.	Maximum.	Minimum.	Change.
0	−100%	0	+200%	0%	200%
40	− 77	0	+177	− 23	200
65	− 42	0	+142	− 58	200
90	0	0	+100	−100	200

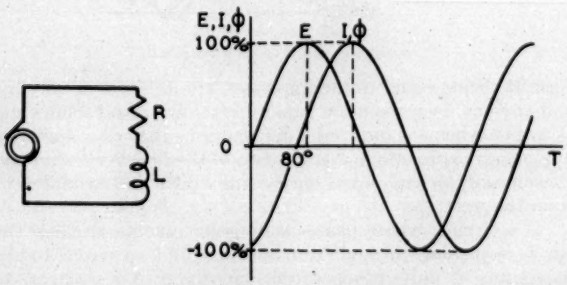
By closing at $wt = 90°$ where the normal flux corresponds exactly with the initial value of the transient flux no energy change is required so there is no transient.

II. AIR-CORED COIL WITH RESISTANCE.

(a) Permanent State.

This is the same as Case I*a* except resistance has been added causing the current and flux to lag 80° behind the impressed e.m.f., instead of 90°, and to decrease 1.5 per cent. in magnitude as compared with that of zero resistance but the maximum of the permanent state is taken as 100 per cent. as shown in Fig. 4.

FIG. 4.



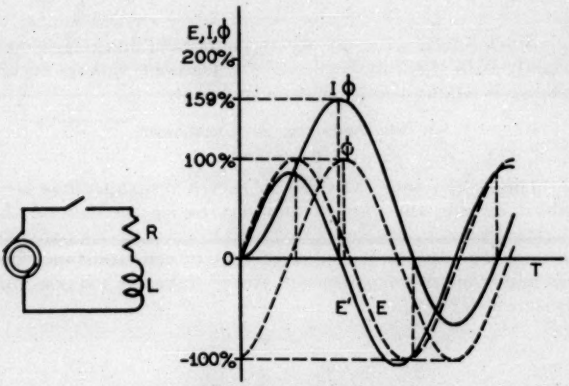
Permanent State for Coil With Resistance.

This circuit may be closed at any time as the resistance will cause the curves to become normal after a few cycles.

(b) Transient State.

Under transient conditions this circuit differs from the one without resistance as follows: while the current is increasing the induced e.m.f., E', is less than the impressed, due to the iR drop, as shown by the solid curve Fig. 5, and as a result the flux and current increase less rapidly than in Case I*b*, even than in Case II*a*, and for a shorter time; while the current is decreasing the induced e.m.f. is larger than in

FIG. 5.



Transient State Closing Circuit at Zero E.M.F.

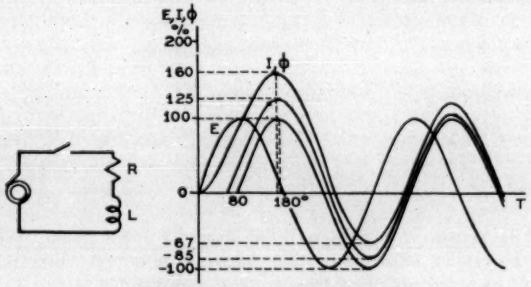
Case I*b*, being equal to the impressed e.m.f. + iR, so the flux and current decrease more rapidly reaching zero earlier, and at a time when the induced e.m.f. requires that they build up in a negative direction. At the end of the first cycle they are considerably negative and succeeding cycles will be different from the first.

If a circuit, whose phase angle is 80°, is closed when the e.m.f. is passing through zero the flux and current reach a maximum of only 159 per cent. at about 160° instead of 200 per cent. at 180° as in Case I*b*. They reach zero at 280°

instead of 360° and a minimum of − 67 per cent. at the end of the first cycle thus approaching the normal state as shown by Fig. 5.

The effect of closing the circuit at different points on the e.m.f. wave is illustrated in Fig. 6.

FIG. 6.



Transient State Closing at Different Points.

The following tabulation emphasizes the principal differences as contrasted with Table I.

TABLE II.

Close at *wt*.	Flux and Current.				
	Normal.	Initial.	Maximum.	Minimum.	Change.
0	−98.48%	0	+159%	− 67%	226%
60	−34.20	0	+125	− 85	210
80	0.00	0	+100	−100	200

It is to be noted that the effect of resistance in reducing the transient is greatest when the circuit is closed where $wt = 0$, as Table II shows and experimentally proved in Fig. 8, and vanishes for wt equal to the phase angle of the circuit, 80° in this case, for at this point the current and therefore the magnetic energy are normally zero and since no change in energy is required, there is no disturbance, that is, the initial transient condition corresponds to the normal at the point of closing the circuit.

In general, for closing at the zero of the e.m.f. wave, $wt = 0$, the value of the maximum transient and the place at which it occurs depends upon the power factor or phase angle of the circuit. For pure inductance it is twice the normal maximum and occurs at the 180° point and in all other cases a few degrees earlier than the normal maximum as shown in Table III.

TABLE III.

Phase Angle.	Maximum Transient.	Occurs At.
90°	200%	180°
80	159	164.0
70	134	152.6
65	126	147.7
60	120	143.2
50	110	135.0
40	105	126.6
30	102	118.0
0	100	90

The reason for this will be evident from what follows. The equation expressing the relation between current and e.m.f., in a circuit of resistance and inductance is

$$L\frac{di}{dt} + iR = E \sin wt,$$

whose solution is

$$i = I \sin (wt - \theta) - I \sin (wt_1 - \theta)\epsilon^{-(R/L)(t-t_1)}.$$

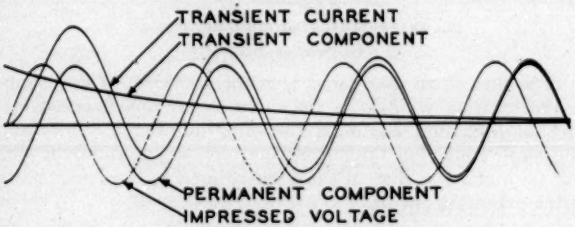
Where "i" is the transient current, $I \sin (wt - \theta)$ the permanent component, $-I \sin (wt_1 - \theta)\epsilon^{-(R/L)(t-t_1)}$ the transient component whose initial value is $-I \sin (wt_1 - \theta) = I'$, "$\theta$" the phase angle of the circuit, "t" time measured from the zero of the e.m.f., t_1 the time when the circuit is closed with reference to the zero of the e.m.f. wave and t' is measured from the instant the circuit is connected.

The permanent or sinusoidal component is independent of the time of closing the circuit; the initial value of the transient component depends solely upon the point of closing the circuit and is equal in magnitude and opposite in direction to the permanent component; and the rate at which the transient component decreases depends only on the ratio, of the resistance to the inductance, the total decrease being a function of the time measured from the instant of closing the circuit.

The maximum transient occurs where the two components have the same sign and slightly to the left of the maximum of the permanent component because it is increasing less rapidly than the transient component is decreasing, in fact, the maximum is reached where the rate of increase of one is equal to the rate of decrease of the other. The transient current reaches its maximum where the permanent component is a maximum for circuits having zero phase angle or 90° phase angle, but for a phase angle of 70° it occurs 7.4° in advance of this position.

By means of the "Transient Visualizer" [3, 4] it is possible to obtain experimentally oscillograms of the transient current and its mathematical components to the same scale and in their proper relation with respect to the impressed voltage, a typical example being reproduced in Fig. 7. This is

FIG. 7.



Oscillogram, Transient Current and Components.

accomplished by taking all of the current curves with a single oscillograph element and making three separate exposures of the film. After each exposure the necessary circuit changes are made and the film exposed again; to be in the proper phase relation, perfect control is necessary.

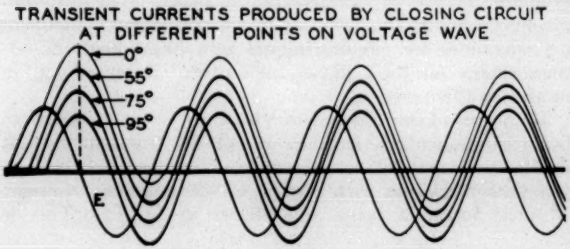
The effect of closing the circuit at different points on the e.m.f. wave is illustrated in Fig. 8 by an oscillogram where successive exposures were made closing at $wt = 0, 55, 75$ and 95°. It is evident the resistance is small as compared to the

[3] H. M. Turner, "The Transient Visualizer," *A. I. E. E.*, 1924.

[4] See Appendix.

inductance as the decrease is very gradual. The circuit consisted of an air-cored coil weighing nearly 200 pounds whose resistance was 4 ohms and inductance 0.5 henry.

FIG. 8.



Oscillogram, Closing at Different Points.

III. TRANSFORMERS WITHOUT RESISTANCE.

(a) Permanent State.

The flux of an iron-cored transformer without resistance in the exciting winding is the same as for an air-cored coil without resistance assuming the same number of turns and impressed e.m.f. but the current required to produce the flux will be much less due to the greatly increased permeability of the magnetic circuit.

(b) Transient State.

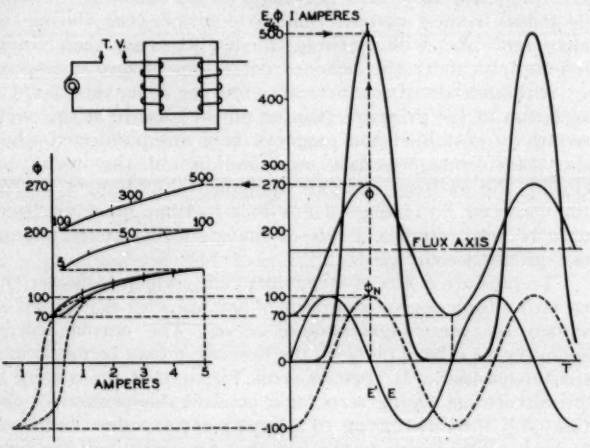
In the assumed case of zero resistance the induced e.m.f. for the transient state is equal to the impressed e.m.f. as before, therefore the flux must change at the same rate, that is, must have the same wave form, as for Case I*b*, but its magnitude will depend not only upon the point on the e.m.f. wave at which the circuit is closed but also upon the magnitude and direction of any residual magnetism in the core of the transformer.

With iron in the magnetic circuit the flux is not directly proportional to the current and since the relation cannot be expressed by any simple equation it is necessary to obtain this information experimentally. The magnetization curve

should be obtained up to about eight or ten times the normal full load of the transformer. It is usually easier to find the flux in per cent. of the maximum of the normal hysteresis loop rather than in lines of force and, for the analysis which follows, is more convenient and just as significant. The performance of a transformer under transient conditions may be predicted with considerable certainty from the observed magnetization data, but even without this information a general understanding of starting currents may be obtained by assuming a reasonable curve.

The magnetization curve of the transformer, used for illustrative purposes, is shown at the left of Fig. 9 in three sections with different current scales, 0 to 5, 5 to 100, and 100 to 500 amperes. The maximum of the normal hysteresis

FIG. 9.



Transformer Without Resistance, Transient State.

loop, shown dotted, is taken as 100 per cent. The current is, in this case, expressed in amperes as a matter of scale convenience while the e.m.f. and flux are expressed in per cent. Note that above five amperes, which is approximately four

times the maximum of the normal exciting current, the ascending and descending curves coincide, but for smaller values the descending branch is higher, due to hysteresis.

The greatest disturbance is produced by connecting a transformer to the line when the e.m.f. is passing through zero provided the residual magnetism is at its maximum, which in ordinary power transformers is often 70 per cent., and the change in flux required by the induced e.m.f. adds to the residual magnetism producing abnormally high saturation of the core and corresponding excessive currents.

Let the circuit be closed when the impressed e.m.f. is passing through zero, increasing in a positive direction, and with a residual flux of + 70 per cent. as indicated in Fig. 9. The actual transient flux is of the same shape as the normal curve already given but in this case, it starts at + 70 per cent. and increases to + 270 per cent. during the first 180° and decreases from + 270 per cent. to + 70 per cent. during the next 180° and will continue to vary between these limits indefinitely under the assumed conditions of zero resistance.

Although the flux starts at + 70 per cent. this is not a violation of the principle pointed out in Case I*b* as the work required to establish this magnetic field was performed when the transformer was last used and not at the instant of closing the switch. A more fundamental statement is that there can be no change of flux in zero time, however, there may be, and often is, a rate of change of flux at the instant the circuit is completed.

To produce a flux of + 270 per cent., which saturates the core to a high degree a current of 500 amperes is required as shown by the magnetization curve. The current corresponding to other points on the flux curve may be found in a similar manner. It appears, from Fig. 9, that the current is uni-directional, being zero for a considerable portion of the time and then increasing to a large positive value, and back to zero. This is due to the fact that for nearly half the time the flux is less than 150 per cent. of the normal maximum requiring less than five amperes which is only 1 per cent. of the maximum shown in Fig. 9, and, on the scale used, scarcely shows being about the width of a line. When the current decreases to zero from large positive values the flux will be

somewhat greater than + 70 per cent. due to hysteresis and a small negative current will be required for a very short interval to reduce it to the initial value to give the necessary 200 per cent. decrease.

For smaller values of residual magnetization the maximum transient current, closing at $e = 0$, will be materially reduced as indicated in Table IV.

TABLE IV.

ϕ_R	+ 70%.	+ 50	+ 30	0	− 30	− 50	− 70
I_{MT}.....	500	340	210	100	30	7	3

The effect of closing the circuit at different points on the e.m.f. wave, for a residual of + 70 per cent., may be quite easily determined from Fig. 9; for example, if the circuit is closed at $wt = 45°$, it may be seen from the normal flux curve that during the remainder of the time the e.m.f. is positive, the flux changes from − 70 per cent. to + 100 per cent. or a total change of 170 per cent., therefore the maximum transient flux is 240 per cent. (70 per cent. residual + 170 per cent. change which in this case is an increase) and the corresponding current is approximate 300 amperes. If the circuit is closed at 90° the change in flux during the remainder of the time the e.m.f. is positive, is from 0 to + 100 per cent. or a total change of 100 per cent. The maximum transient flux is 170 per cent. and the corresponding current 30 amperes. The results are given in Table V.

TABLE V.

wt.	0	45	90	135	180	225	270	315	360°
I_{MT}....	+500	+300	+30	+1.2	−3	−1.2	+30	+300	+500

Briefly, for a transformer without resistance the impressed e.m.f. determines the induced e.m.f. and hence the rate of change, or wave form, of the flux wave whose magnitude is determined by the residual and the point on the e.m.f. wave at which the circuit is closed. The flux and magnetization curve determine the current.

IV. TRANSFORMERS WITH RESISTANCE.

(a) Permanent State.

The normal flux in an unloaded transformer with resistance is almost exactly the same as for one without since the iR drop due to the normal exciting current is usually extremely small producing a negligible decrease in the induced e.m.f. and would therefore require the same flux.

(b) Transient State.

In the transient state, the actual flux during the first 60° is practically the same even under the most adverse condition of a large residual and closing at zero e.m.f., for a transformer with resistance as for one without for the current required to give the accompanying increase in flux will be small as compared to full load, the iR drop is negligible and the induced e.m.f. is equal to the impressed up to about 60° as in Case III*b*. However, beyond this point the current increases rapidly as the core becomes saturated and the iR drop diminishes appreciably the induced e.m.f. and the flux required to produce it.

By definition the induced e.m.f. may be expressed by

$$e = n\frac{d\phi}{dt} \quad \text{from which} \quad \phi = \frac{1}{n}\int_0^t e dt,$$

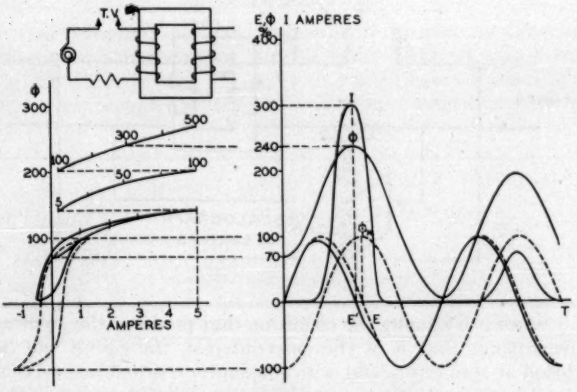
in other words, the change in flux is proportional to the area under the induced e.m.f. curve up to the time, t, and since the iR drop diminishes this area the flux is less than for a transformer without resistance as indicated in Fig. 10.

As a result of the lower flux density the current, although more than six times full load, is much less than for the assumed case of zero resistance. While the current is increasing the induced e.m.f. is less than the impressed due to the iR drop, and as a result, the flux and current increase less rapidly and for a shorter time, reaching a maximum of 240 per cent. and 320 amperes respectively at 165° in this case, instead of 270 per cent. and 500 amperes at 180° for zero resistance. While the current is decreasing the induced e.m.f. is greater than the impressed, due to the iR drop, so the flux and current decrease more rapidly, approaching zero at 260° instead of at 360°, and from this point on to 450° the current is small

compared to the maximum due to the low flux density. For a short time the current is actually in negative direction but of too small magnitude to show on the scale used.

Since the flux increases less rapidly and for a shorter time and decreases more rapidly and for a longer time, than for the case of zero resistance, it is evident that the flux at the end of the first cycle will be considerably less than at the beginning, thus nearer the normal condition: however, after

FIG. 10.



A 5-kw. Transformer. Total Primary Resistance 0.1.

the first cycle the current will be much smaller, due to lower saturation, and the effect of the iR drop will be correspondingly reduced so the approach to normal will be more gradual, requiring several cycles for the current and flux to become symmetrical with respect to the time axis.

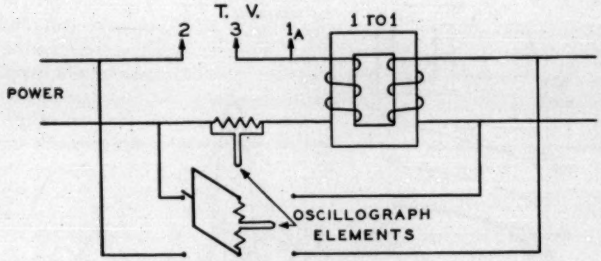
The conclusions deduced analytically from the magnetization curve of one transformer are confirmed experimentally by oscillograms taken on another. Since it is the principles involved rather than the performance of a given transformer, the size is of no importance here. To avoid drawing large currents from the power system with consequent voltage fluctuation the oscillograms reproduced in this paper were

taken on a 600 watt transformer, however, similar curves have been obtained on both larger and smaller sizes.

EXPERIMENTAL PROCEDURE.

The circuit used is shown in Fig. 11. "T.V." is the abbreviation for the "transient visualizer," briefly described in the Appendix, its function being to close and open the circuit at desired points.

FIG. 11.



Circuit Used for Obtaining Oscillograms.

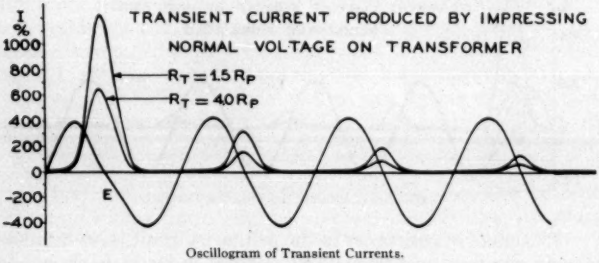
Since it is usually the condition that produces the greatest disturbance that is of the most interest, the circuit will be closed at zero e.m.f. and with maximum residual magnetism, in all but one or two cases which are easily identified. The experimental procedure is as follows: To locate the zero of the impressed e.m.f. disconnect the primary of the transformer and replace it by a non-inductive test circuit including an oscillograph element in series, or the voltage element shown in Fig. 11 may be used by shifting the lead from terminal 2 of the transient visualizer to terminal 3, and with the transient visualizer at synchronous speed observe, on the screen of the oscillograph, a heavy ground line up to the point where the test circuit is closed. Due to the persistence of vision one will also see a complete voltage curve produced by the same element during the previous revolutions. Rotate the main brush arm until the break in the ground line coincides with the zero of the e.m.f. wave and lock the arm in position.

Reconnect the primary and rotate brush 1*a* until the transient current is a maximum, which indicates that the residual has its greatest value and correct polarity. Should it be necessary to stop the transient visualizer before all of the desired curves are obtained, in re-starting it the motor may pull in so the circuit is closed at the point for which it was adjusted or 180° out of phase. This may be easily corrected by re-starting the motor, usually once or twice, until the phase is correct.

OSCILLOGRAPHIC RESULTS.

The transient current produced by impressing normal voltage on a transformer may be quite large, as shown by Fig. 12 where it is of the order of twelve times normal full load even though the resistance has been increased 50 per

FIG. 12.



Oscillogram of Transient Currents.

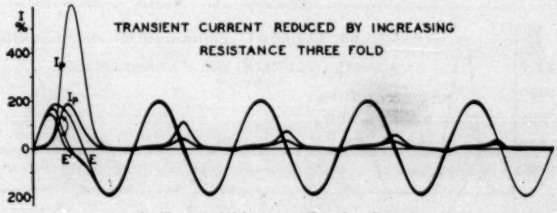
cent. or really the equivalent of a larger increase since there was a line drop as indicated by the voltage curve. Increasing the resistance to about four times that of the primary winding reduced the current to approximately half value. The elements involved will be discussed in connection with Figs. 13 and 14 where more information is available.

To limit the current, so that the continuous repetition of the transient over a considerable period of time will not produce excessive heating, the resistance of the circuit is increased to approximately six times that of the primary winding alone in all cases except where otherwise noted.

EFFECT OF RESISTANCE ON TRANSIENT.

Since the area under the induced e.m.f. curve is a measure of the change in flux, it is important to obtain this curve to the same scale as the impressed e.m.f. which is accomplished by using a one to one ratio of transformation, and, after taking an oscillogram of the impressed wave, the voltage element with its series resistance is connected across the secondary and the film exposed a second time superposing the induced e.m.f., under transient conditions, upon the impressed. Fig. 13 shows the impressed and induced voltages and the transient current for six and eighteen times the resistance of the primary winding. Actually all five curves were taken with a single element by making five separate exposures of the film. As a matter of convenience 110 volts is impressed upon the 100 volt winding.

FIG. 13.



Oscillogram of Voltages and Transient Current.

The effect of resistance in the primary circuit is pronounced during the first cycle when the current is large as shown by the difference between the induced and impressed voltages. As previously shown for the ideal case of zero resistance, the induced e.m.f. coincides with the impressed e.m.f. and, for closing at the zero point of the e.m.f. wave, the total change in flux is 200 per cent. Considering the upper curve of Fig. 13, the area under the induced e.m.f. is about 0.6 of that under the impressed which corresponds to a change in flux of only 120 per cent., 0.6 of 200 per cent., and a maximum transient flux of 190 per cent., 70 per cent. residual (assumed) + 120 per cent. increase. As a result the current reaches its maximum earlier, at about 135°, and for this reason is much

smaller in magnitude being about half that in Fig. 12. The maximum current occurs where the induced e.m.f. is zero and at this point the impressed voltage is all absorbed by the resistance. The area under the negative half cycle of the induced e.m.f. is approximately 12.5 per cent. greater than that under the impressed so the flux decreases 225 per cent., 1.125 times 200 per cent., or from + 190 per cent. to − 35 per cent. The negative flux requires a negative current but for this curve it is so small that it scarcely shows on the oscillogram. At the beginning of the first cycle the flux was 170 per cent. above normal, for it started with a residual of + 70 per cent. while its normal value at this point is − 100 per cent. At the beginning of the second cycle it is only 65 per cent. above normal, which accounts for the much smaller value of current at the second maximum. From here on the approach to the normal condition is gradual for the current and iR drop are relatively small so there is little difference between the induced and impressed voltages.

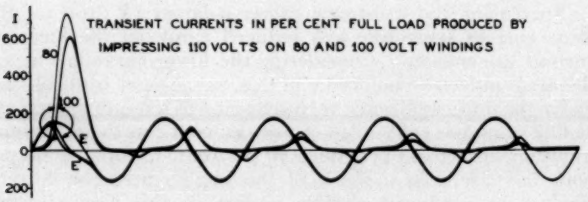
Increasing the resistance causes a larger iR drop for the same current, therefore less induced e.m.f. so the flux and current are smaller. Considering the lower curve in Fig. 13, the area under the induced e.m.f. is, say, 45 per cent. of that under the impressed wave corresponding to a required increase of flux of 90 per cent., 0.45 of 200 per cent., so the maximum transient flux is 160 per cent., 70 per cent. residual + 90 per cent. increase. As a result of the smaller area the current reaches its maximum slightly earlier, in this case at 120°, and its value is only 32 per cent. that of the other curve, but as the resistance is three and a half times greater the iR drop is approximately 12 per cent. higher which is due to the fact that the maximum occurs earlier, therefore, at a time when the impressed e.m.f. has a higher value. However, this increase in voltage is always less than the increase in resistance so the current maximum is reduced. When the current is a maximum all of the impressed voltage is consumed by the resistance and $i_{max.} = e/R$. However, if the resistance is increased n fold the maximum occurs earlier where the voltage is slightly greater, therefore $i_{max.}$ is slightly greater than $1/n$th of its former value. The negative area is approximately the same as before so the flux decreases 225 per cent., or

from 160 per cent. to − 85 per cent., requiring a slightly larger negative current, as shown in Fig. 13.

The Effect of Saturation on Transient.

In Fig. 14 are shown two sets of curves resulting from impressing 110 volts upon the 100 volt winding, as in Fig. 13; and the 80 volt winding. Superposing the curves in this way makes it easier to compare the results. With the higher saturation the current will necessarily increase more rapidly and reach a higher maximum, for a greater change in flux is required. Referring to Fig. 14 it will be observed that the area under the induced e.m.f. curve is less for the 80 volt winding than for the 100 but this does not mean that the actual flux increase is less, but only a smaller percentage increase. The maximum current is larger for two reasons: first, the resistance is less due to using fewer turns; and second,

FIG. 14.



Oscillogram Showing Effect of Saturation.

the voltage is higher because the maximum is reached earlier. As a result of the larger current the effect of resistance is more pronounced. Since the action in the case of the 100 volt winding has been analyzed it will not be repeated here but will be used as a reference standard. For the ideal case of zero resistance and closing at zero e.m.f. a 200 per cent. change in flux is required during the time the impressed e.m.f. is positive. Under the same condition the 80 volt winding would require 25 per cent. more flux, to compensate for the fewer turns, or an increase of 250 per cent., however, in the actual transformer with resistance the area under the induced e.m.f. is approximately 0.5 of the ideal, therefore,

the increase is only 125 per cent. Due to higher saturation the residual has increased somewhat, say to 75 per cent., which brings the maximum up to 200 per cent. If the area under the negative half cycle be taken as 12.5 per cent. greater than for the ideal case of zero resistance then the decrease in flux will be 280 per cent., 1.125 times 250 per cent., so at the end of the first cycle the flux will be − 80 per cent. However, with the higher voltage per turn it must be kept in mind that the normal flux is − 125 per cent. Thus it is seen that the approach to normal is much more rapid at first than for the 100 volt winding, in fact, the effect is so exaggerated as to cause appreciable negative current during the first cycle and, after the second, the curve is almost symmetrical.

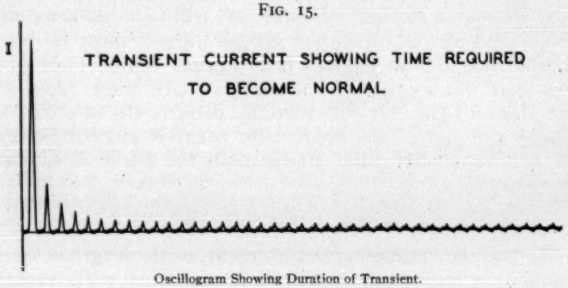
DURATION OF THE TRANSIENT.

To find the duration of the transient, or the length of time required for the current to become symmetrical with respect to the time axis, requires a slightly different procedure: The transient visualizer and photographic drum are usually driven at 600 r.p.m. so the circuit is closed for less than 0.1 second, corresponding to six cycles of power frequency, which is not sufficient for the current to become normal, however, it is not feasible to reduce the driving speed appreciably for the decreased mechanical displacement corresponding to a given number of electrical degrees would make the brush setting less precise. This difficulty may be avoided by driving the photographic drum and a second transient visualizer at, say, 60 r.p.m. or any other desired speed, closing the transformer circuit through brushes two and three of the first transient visualizer running at 600 r.p.m. in the usual way, and maintaining the circuit closed for the desired time through brushes two and three of the second transient visualizer, and, finally transferring the control back to brush 1*a* of the first machine so the circuit may be opened at a time when the residual magnetism is of the desired magnitude and polarity. This arrangement may be effectively applied to the solution of other problems.

The oscillogram reproduced in Fig. 15 is for closing the circuit when the e.m.f. is passing through zero increasing in

a positive direction and with a maximum positive residual. After a few cycles the approach to normal is very gradual and at least forty cycles are required for the current to become approximately symmetrical with respect to the time axis, and many more before it is actually symmetrical.

FIG. 15.

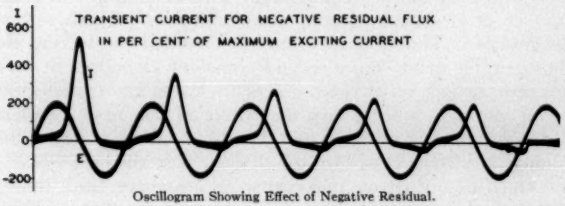


Oscillogram Showing Duration of Transient.

The Effect of Negative Residual Magnetism.

In the cases so far considered the initial change in flux added to the residual magnetism often producing high saturation and large current. With negative residual and

FIG. 16.



Oscillogram Showing Effect of Negative Residual.

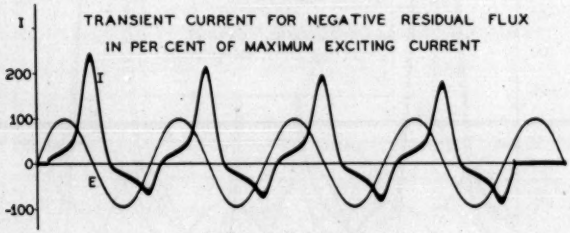
the circuit closed at zero e.m.f. the initial change in flux reduces the residual and the saturation is much less so it is desirable to express the transient current in terms of the maximum normal exciting current as shown in Fig. 16.

With a negative residual of approximately 60 per cent., the first positive maximum will be of the order of 140 per cent.,

requiring between five and six times the normal exciting current. On account of the small current the iR drop will have only a slight influence in reducing the current to normal and the approach is much more gradual than in Fig. 15. It will be observed that the current reaches a maximum at 180°. Neglecting the effect of resistance the flux, at the end of the first cycle would be − 60 per cent. but taking the resistance into account it would be slightly lower, say − 65 per cent., requiring a negative current that shows on the oscillogram.

By closing 25° later the change in flux required during the remainder of the time the e.m.f. is positive is 190 per cent. and starting with − 60 per cent. the flux builds up to + 130 per cent. at the first maximum requiring about two and a half times the normal exciting current. Referring to Fig. 17 it will be seen that the approach to normal is still more gradual.

FIG. 17.



Oscillogram, Closing at 25°.

V. TRANSIENT CURRENTS IN LOADED TRANSFORMERS.

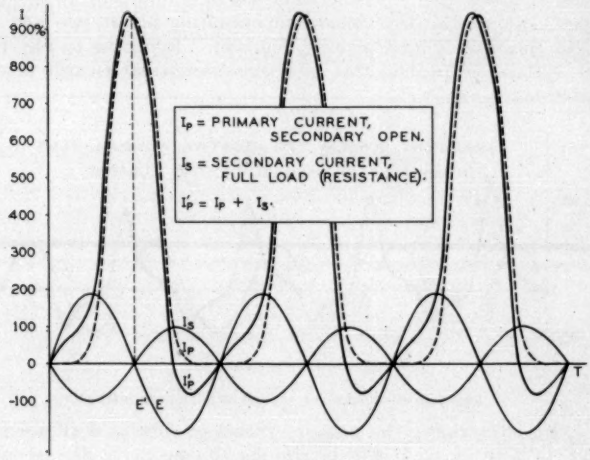
In determining the primary transient current with a load on the secondary, it will be simpler to consider a one to one transformer having negligible leakage in the discussion which follows, and in the preliminary analysis of each type of load, it will be further assumed that the transformer windings are without resistance.

(a) Resistance Load.

In such a transformer the induced voltage in both primary and secondary is always equal to the impressed regardless of

the conditions of operation, that is, whether the transformer is loaded or unloaded, or in the transient or permanent state. The load voltage is the same as that impressed upon the primary. With a resistance load on the secondary and the primary closed to produce the maximum transient disturbance, a most interesting condition arises: the secondary is normal from the instant of closing the switch and the primary never becomes normal under the assumed conditions. The primary current consists of two components: the transient that would exist with the secondary open, shown dotted in Fig. 18, and a load component equal in magnitude and

FIG. 18.



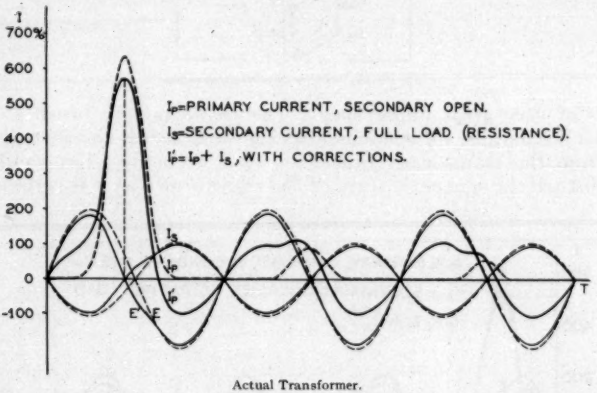
Ideal Transformer, Resistance Load.

opposite in direction to the secondary current. On account of the phase relation the maximum transient current with load is almost exactly the same as without load. However, there is a negative current for a small part of each cycle.

When the windings have resistance the problem is somewhat more complicated; however, as a first approximation add the normal load component to the no-load transient

curve. The result is in error for two reasons: first, when the primary no-load transient current is large the induced e.m.f. is modified by the iR drop, resulting in the secondary current being distorted as shown by the solid curve in Fig. 19; second, the load component causes additional iR drop in the primary thus further modifying the induced e.m.f. Therefore, if the two components be added the resulting curve will be slightly above I_p, up to the maximum, and the resulting larger iR drop decreases the induced e.m.f. thus diminishing the required increase in flux. With the lower saturation the current and iR drop will decrease thus partially annulling the tendency towards lower values but in any event the

FIG. 19.



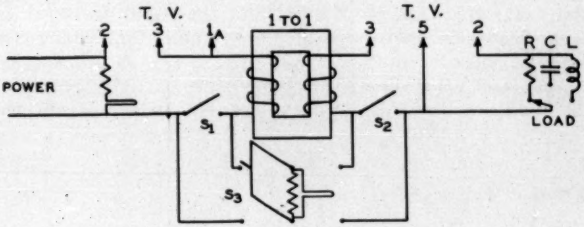
Actual Transformer.

maximum flux and current will be less than for the no-load transient on account of the fact that with load the iR drop is appreciably larger during the first 90° or 100° thus reducing the flux increment and causing the transient current with load to cross that without load before reaching a maximum. The third approximation gives the solid curve I_p' in Fig. 19 which is quite close to the actual curve.

In order to verify these conclusions experimentally the circuit shown in Fig. 20 is used. The load is connected to

the secondary, by a transient visualizer, immediately before connecting the primary to the line and is opened first so there will be no current in the secondary after the primary is opened. This makes it possible to control the residual magnetism by brush 1*a* of the primary transient visualizer. This

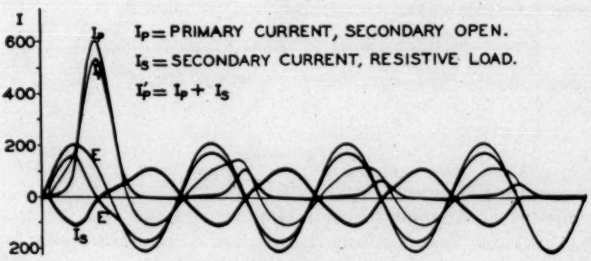
FIG. 20.



The Circuit Used for Load Oscillograms.

is of very great importance. The connection to brush 5 is for discharging the condenser during the time it is disconnected from the transformer otherwise when reconnected it would disturb the magnetic state of the transformer as it is periodi-

FIG. 21.



Oscillogram for Resistance Load.

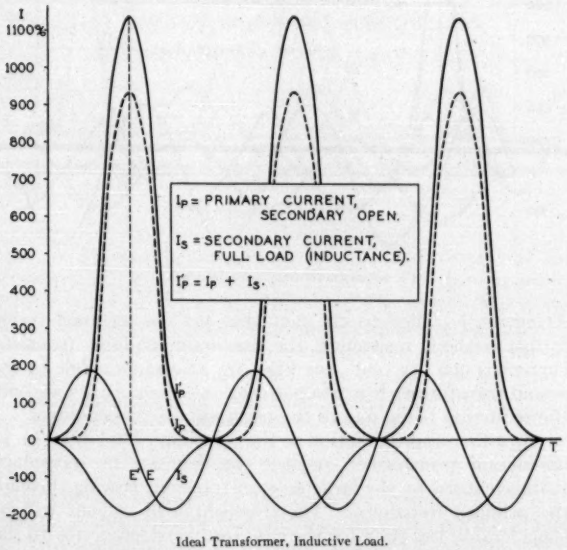
cally connected and disconnected by the transient visualizer. The primary and secondary currents are taken with the same oscillograph element and are, therefore, to the same scale. Also the voltages are taken by the same element by the switching arrangement shown in Fig. 11.

Oscillographic curves of the no-load transient, the primary and secondary transient currents with a resistance load and the primary and secondary voltages are reproduced in Fig. 21 making it possible to study the results in detail. The large difference between the voltages is due to the fact that there is six times normal resistance in the primary.

(b) Inductive Load.

In a one to one transformer, without resistance in either winding, the load voltage is the same as that impressed on the primary regardless of operating conditions. The secondary transient current is found as outlined in Case I*b* and shown in Fig. 2. The primary current is found by adding to the no-load transient, the dotted curve in Fig. 22, the load component.

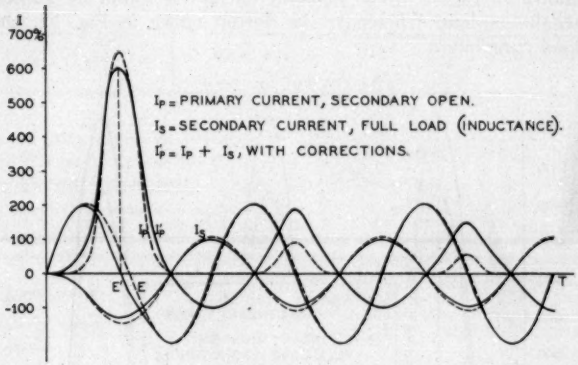
FIG. 22.



Ideal Transformer, Inductive Load.

If the circuit is closed at zero e.m.f. and with maximum positive residual, neither the primary nor the secondary currents will ever be symmetrical with respect to the time axis but are uni-directional. The secondary current, which is always 180° out of phase with the primary, has a maximum of twice the normal full load at 180°, so the load component adds directly to the no-load transient causing an appreciable increase in the primary current due to the demagnetizing action of the secondary. Had the circuit been closed at the maximum e.m.f. the secondary current would have been normal and the primary abnormal but of smaller value.

FIG. 23.



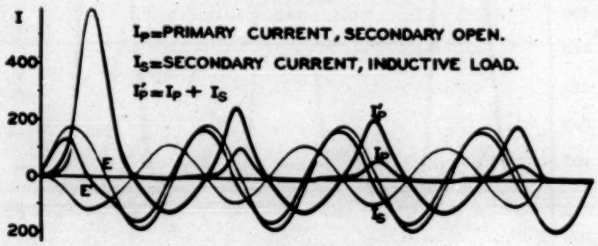
Actual Transformer, Inductive Load.

Attention is called to the fact that for the assumed transformer without resistance, the maximum no-load transient current is 900 per cent., see Figs. 18, 22 and 25, while in the actual transformer it is 630 per cent., see Figs. 19, 23 and 26, the reduction being due to the resistance of the windings.

As a first approximation to the secondary load current in the actual transformer, neglect the decrease in secondary voltage caused by the large no-load transient flowing through the primary resistance. The transient load current will be considerably less than for the case just considered, due to the

secondary resistance, and is represented by the dotted curve in Fig. 23. Next correct for the iR drop in the primary due to the no-load transient current; the result is indicated by the solid curve. As a first approximation to the primary current, add the load component just found to the no-load transient. This curve, not shown, will establish an upper limit which is obviously too high for the resulting current will be above the no-load transient up to its maximum, thus increasing the iR drop and decreasing the induced e.m.f. and the maximum flux. However, correcting for the increased iR drop and from the magnetization data finding a new current curve, not shown, a lower limit is established which is too low, for with a smaller current the tendency of the induced

FIG. 24.



Oscillogram for Inductive Load.

e.m.f. to decrease is partially annulled. The corrections are necessary only near the maximum point. The third approximation is shown by the solid curve in Fig. 23 which is quite similar in shape to the oscillogram reproduced in Fig. 24.

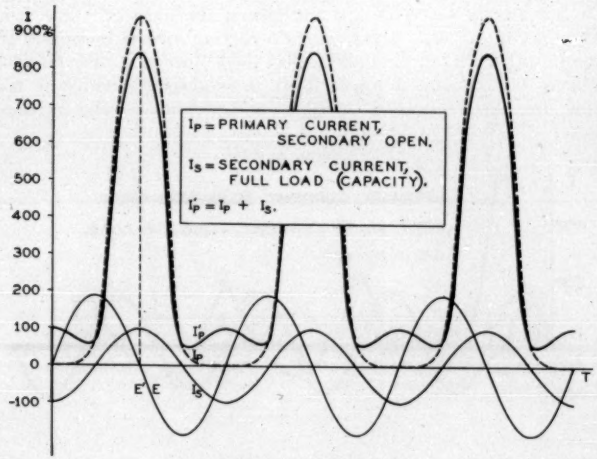
From the oscillogram it is seen that the primary current up to 270° is almost exactly the same whether the secondary is loaded or not. The difference between the primary and secondary voltage is quite marked.

(c) Capacity Load.

Where the transformer windings are assumed to have no resistance the primary current may be obtained by adding the two components. If the circuit is closed at zero e.m.f.

and with maximum residual the secondary load current will be normal and the phase such as to reduce the maximum primary current. The reason for this is that from 90° to 270° the load current flowing through the secondary winding produces a magnetizing effect in phase with that of the primary thus reducing the primary current, instead of in-

FIG. 25.



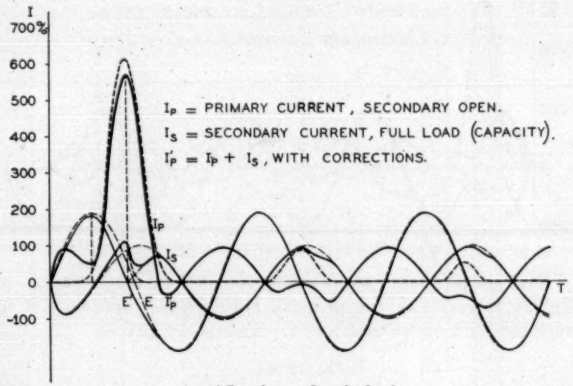
Ideal Transformer, Capacity Load.

creasing it as in the case of inductance. From 0 to 90° and from 270° to 360° the secondary current is demagnetizing thus requiring a larger primary current, but this is where the no-load transient current is very small. For a pure inductive load the secondary current was demagnetizing at all times thus requiring a larger primary current.

In an actual transformer the resistance of the windings will modify these curves. The secondary resistance will cause the current to lead the e.m.f. by less than 90°, 80° will be used in the following discussion. Assume that the circuit is closed at zero e.m.f. and with a maximum positive

residual. As a first approximation to the secondary load current neglect the iR drop in the primary due to I_p and the dotted curve is obtained. This curve is normal after 90°. Next correct for the primary drop. Under normal conditions the induced e.m.f. in the secondary would be practically identical with the dotted curve, E. At 100°, or thereabouts, the condenser is fully charged and begins to discharge against an e.m.f., E', that is less than normal, therefore the condenser will discharge at a higher rate as indicated by the solid curve. At a point where the induced e.m.f. is zero all of the voltage of the condenser is consumed by the resistance of the secondary windings. If there was no other e.m.f. in the circuit the

FIG. 26.

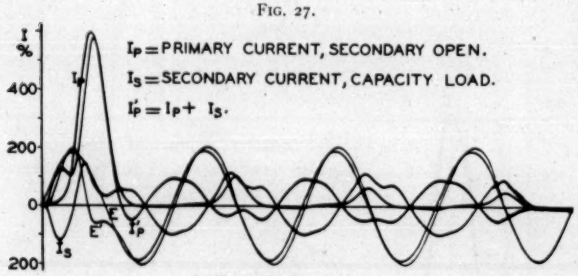


Actual Transformer, Capacity Load.

condenser would discharge very rapidly as shown by the dotted curve. However, the induced e.m.f. has changed direction and is increasing in magnitude and if it were the only e.m.f. present the current would build up as shown by the second dotted curve. Actually both of these effects are taking place at the same time so the current decreases less rapidly due to the induced e.m.f. aiding the condenser e.m.f. and as the induced e.m.f. becomes larger the current actually increases. Very soon the condenser is completely discharged

and immediately charges in the opposite direction. The load current is shown by the solid curve. As a first approximation to the primary current, add the load component just found to the no-load transient and there results a curve, not shown, that is too low. The reason for this is that for the smaller current, the iR drop would be less than for the no-load transient alone, so the induced e.m.f. will increase and will require a larger flux and, as a result, the primary current will increase to some extent. The third approximation gives the solid curve shown in Fig. 26 which compares favorably with the oscillogram reproduced in Fig. 27.

FIG. 27.



Oscillogram for Capacity Load.

The oscillogram shows that the secondary voltage is slightly larger than the primary between 80° and 90°; it is constant for a short time after it becomes negative.

APPENDIX.

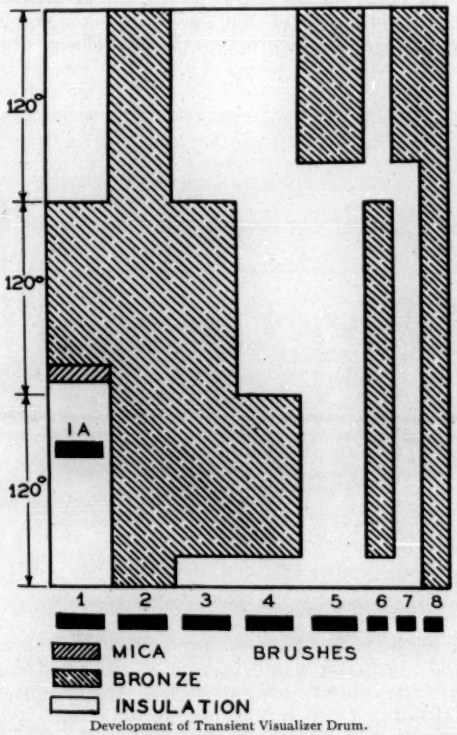
TRANSIENT VISUALIZER.[5]

The "transient visualizer" is a synchronous switch for controlling circuit conditions in connection with oscillographic studies of periodic and transient electric phenomena. It was so named because it makes possible visual observation of transient curves on the screen of an oscillograph. This is accomplished by repeating the phenomena synchronously with respect to the period of the oscillating or rotating oscillograph mirror thus making the transient periodic. It

[5] H. M. Turner, "The Transient Visualizer," A. I. E. E., 1924.

consists essentially of a four-pole, synchronous motor connected through helical gears to an insulated drum into which contact segments have been set, 1 to 5 inclusive being electrically connected internally as are 6, 7 and 8 but the two

FIG. 28.

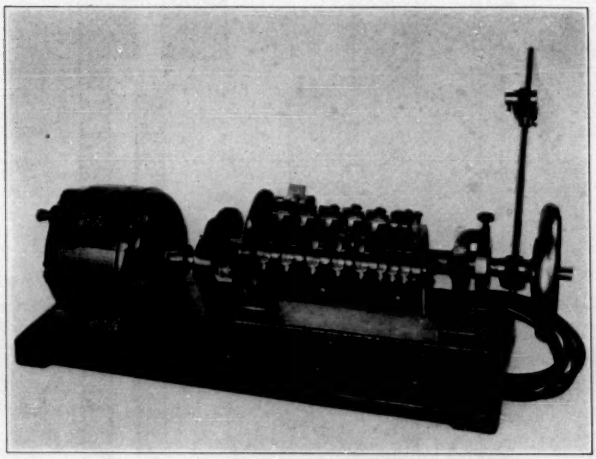


Development of Transient Visualizer Drum.

groups are insulated from each other. The lengths and relative angular positions of the various segments, shown in a development of the drum in Fig. 28, are designed to give maximum flexibility in its use.

Brushes 1 to 8, which make contact with their respective segments, are mounted on a common arm and may be rotated through a large angle and locked in any desired position in order that the curves may be properly placed on the visual screen or on the photographic film and the circuit closed at any desired point on the e.m.f. wave. Attached to the arm through gears is a pointer that moves over a scale graduated in electrical degrees which makes possible closing the circuit

FIG. 29.



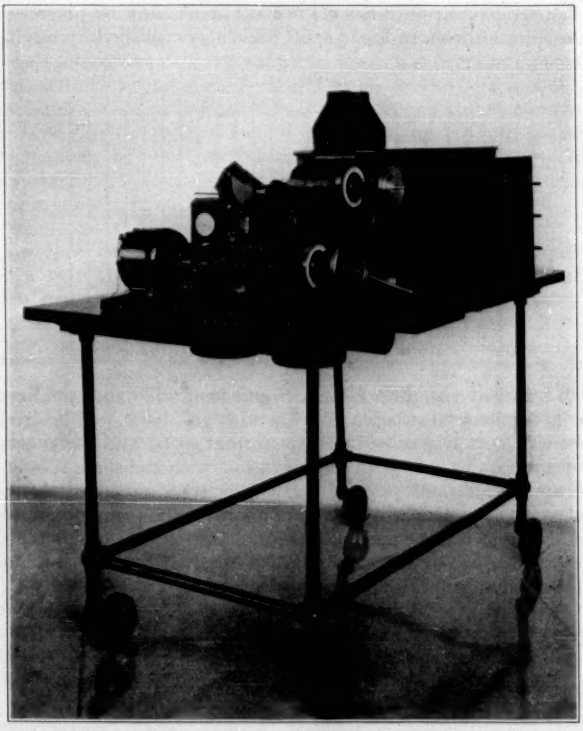
Transient Visualizer.

at definite points on the e.m.f. wave. Brush 1*a*, mounted on a disk at the left of Fig. 29, may be rotated independently of the main brush arm and is usually connected in parallel with one of the other brushes to increase the time the circuit remains closed or to open it at any desired point. Observing the curves on the screen greatly facilitates preliminary adjustments.

For obtaining photographic records the transient visualizer is mounted in front of the oscillograph cabinet, as shown in

Fig. 30, and is connected to the photographic drum by a sprocket and steel chain drive free from lost motion making it possible to retrace any desired curve as many times as necessary to obtain the required line density. For the purpose of comparison a series of curves of related phenomena may be taken in succession on the same film. The circuit

FIG. 30.

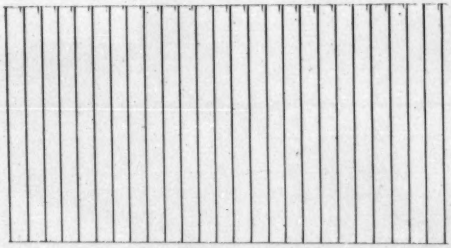


Transient Visualizer and Oscillograph Assembly.

may be closed at the same point of the e.m.f. wave or at different points. In any case an exposure is made and then the conditions are changed and a second exposure is made, the operation being repeated until all the curves have been taken.

If the power frequency is 60 cycles the transient visualizer may be used to place a time calibration on a film. By using a worm drive between the transient visualizer and the photographic drum, lines 0.1 second apart may be placed on the film as shown in Fig. 31. This is accomplished by having

FIG. 31.



the transient visualizer close a circuit that will cause the light from a mirror to sweep across the film and back. If desired, the oscillogram may be taken at normal speed and the timing lines put on at low speed as shown but with the same spacing to represent .001 second. By selecting suitable circuit constants any weight of line from a faint trace to heavy lines, as shown in Fig. 31 may be produced.

Special uses of the device are illustrated by experimental results in appropriate places in this paper.

PARAGUTTA, A NEW INSULATING MATERIAL FOR SUBMARINE CABLES.

BY

A. R. KEMP,

Bell Telephone Laboratories.

Formerly deep sea cables were used exclusively for telegraph purposes but in recent years there has been an increasing use of this type of cable for telephone service. Telephonic communication requires cables of very much superior transmission quality to that needed for telegraphs. At the higher frequencies of voice transmission the energy losses in the insulating material become a serious factor and a radical improvement in submarine insulation is called for.

The longest existing deep sea cables operating at voice frequency only slightly exceed 100 miles and the construction of a transoceanic telephone cable with standard materials has been regarded as beyond the practical limits of feasibility.

The installation and rapid expansion of transatlantic radio telephony during the past few years has created a need for a deep sea telephone cable to supplement this service particularly during periods of atmospheric disturbances. In addition the development of carrier telephony offers possibilities for increasing the traffic over shorter submarine cables. For the shorter cables, the still higher frequencies of carrier telephony make demands upon the insulating material similar to those of long cables operating at voice frequency.

In view of these circumstances an extended study was undertaken of the causes of losses and other electrical weaknesses of submarine insulation and a search has been made for better materials. As a result of this investigation an insulation called paragutta has been developed which as the name suggests is derived essentially from rubber and gutta percha. It is the purpose of this paper to describe this material and give an account of the tests to which it has been subjected to determine its suitability for the purpose.

By virtue of its superior electrical properties, the use of paragutta in place of gutta percha for the insulation of telephone and telegraph cables also offers advantages either from the standpoint of improved transmission or the economies in materials of construction which can be made as a result of modified design.

Gutta percha and balata have been the standard materials for the insulation of deep sea cables since the inception of the submarine cable industry some seventy-five years ago. Although these substances are inadequate for modern telephone needs as regards their electrical characteristics, their mechanical properties are peculiarly adapted to submarine insulation. This is so much the case that gutta percha can fairly be taken as a model which must be closely imitated in respect to mechanical characteristics by any successful substitute. This is fortunate since the use of any substitute which differs radically from gutta percha would mean discarding large existing investments in special technique, equipment and trained personnel, and would involve serious risks as to the integrity of cables made with the new material. It may be remarked in passing that no manufacturing process requires a higher degree of insurance against occasional defects than does the submarine insulation art, a fact that has engendered a strong conservatism in the industry.

Because of its almost ideal mechanical properties, the requirements for submarine cable insulation may conveniently be described by reference to gutta percha. Gutta percha insulation, which often includes more or less balata in its composition, is made of raw materials carefully selected for quality, which are thoroughly washed and extremely uniformly blended. The thermoplasticity of the material is of great service in these operations and further permits it to be readily extruded onto a conductor in multiple layers in a continuous sheath with great exactness and freedom from mechanical defects. After being forced around the conductor the material quickly sets to a hard, tough covering when drawn through cold water. Its firmness and toughness are essential to resist subsequent handling operations in the factory, as well as those involved in laying, picking up and repairing. The warm, soft material adheres readily to the conductor and is well adapted

to the making of joints in the insulation between core lengths both in the factory and on the cable ship.

In addition to these excellent and unique mechanical properties, gutta percha possesses electrical characteristics peculiarly adapted to submarine cable construction. Its outstanding electrical merit consists in the fact that its electrical characteristics are stable under sea bottom conditions over a great many years.

Gutta percha is obtained from the latex of a large number of species of trees growing wild in the forests of the Malay Peninsula and the East Indian Islands. The products of the various species of trees are by no means of equal value, varying as they do in the content of hydrocarbon, resins, moisture and other substances. Since the material is gathered and worked up upon the spot by primitive people a great deal of carelessness as well as deliberate adulteration is practiced and the material comes upon the market in a dirty condition and in a bewildering variety of forms which almost prohibit effective inspection, standardization and grading.

The essential constituent of gutta percha is an unsaturated hydrocarbon of colloidal nature which is similar in its chemistry to rubber. It is this constituent which makes gutta percha plastic when warm and tough when cold, and which contributes most conspicuously to its electrical excellence as an insulator. The usual gutta percha insulation is the result of blending and washing various grades of crude gutta percha to remove dirt and water soluble components. The hydrocarbon, resin, dirt and moisture contents as determined by analysis of the crude material together with the electrical and mechanical properties after washing are the principal characteristics used to determine whether or not a particular grade of crude gutta percha is suitable for use as submarine cable insulation. The hydrocarbon content of gutta percha insulation when applied to the conductor is usually about 60 per cent., the remainder being mostly the natural resins together with small amounts of very finely divided dirt (humus) and residual moisture. The proteins or albumens in crude gutta percha and balata are almost completely removed by simple washing.

Balata comes from two species of trees of the same general botanical family as gutta percha, but is native to the forest regions of upper South America and is unknown in the gutta percha producing area of the Far East. The latex of the balata tree is more fluid than that of gutta percha, which permits the trees to be tapped and the fluid to be collected at a central point in the forest, where the product from various trees is mixed for recovery of the gum. Because of the small number of species involved and the transportability of the fluid latex, balata is produced in a much more limited number of grades and is cleaner and more dependable as to uniformity of quality. Its essential constituent is the same hydrocarbon which gutta percha contains. In addition to the hydrocarbon there is present in balata some 40 per cent. of resins and amounts of dirt, moisture and other impurities which usually total about 15 per cent. The resins of balata are softer than those of gutta percha and make the product in its raw state a little less desirable than the better grades of gutta percha from the mechanical standpoint. Balata, however, contains a smaller amount of finely divided dirt or humus than gutta percha which is reflected in its superior electrical characteristics and lower water absorption.

The resins of both gums have been usually included with the hydrocarbon in making submarine insulation. Sometimes, however, a portion of the resins are removed, partly to increase the toughness and partly to improve the electrical characteristics.

There are several methods which may be used for preparing gutta hydrocarbon nearly free from resinous substances. One of these methods involves dissolving the balata or gutta percha in warm petroleum naphtha, filtering the solution from dirt and precipitating the gutta hydrocarbon from solution by refrigeration leaving most of the resins in solution. A simpler and less expensive method, however, is that of leaching out the resins by simply soaking the sheeted or finely cut material in a suitable grade of petroleum naphtha at ordinary temperature followed by draining off the solution of resins and finally evaporating the residual solvent from the extracted material.

The completely deresinated hydrocarbon from either source is not suitable for use alone as submarine cable insulation because insufficiently plastic at safe working temperatures, as well as prohibitively expensive. Otherwise the complete deresination of these products would be highly advantageous as for example is indicated by the superior electrical characteristics of deresinated balata shown in Table I. A substantial amount of experimentation upon the methods of refining balata has been necessary to secure the excellent electrical characteristics therein indicated but no revolutionary innovation has been necessary.

TABLE I.

Effect of Resin Content on the Electrical Characteristics of Balata.

	Electrical Characteristics 0° C., 1 Atm., 2000 Cycles.	
Material.	Dielectric Constant.	Specific Conductance Unit = 10^{-13} mho. cm.
Balata	3.1	66
Deresinated Balata	2.6	3
Balata Resins	3.3	52

In attempting to develop a new insulating material for deep sea cables it seemed best to begin with gutta hydrocarbon as a basis, since its mechanical properties are so unique, rather than to attempt to synthesize a new chemical compound which would imitate it. In order to overcome the excessive stiffness of the pure gutta hydrocarbon, as well as its prohibitive cost, it was determined to attempt to blend large quantities of rubber with it, since rubber is the nearest kindred material and is commercially available at low cost. There resulted thermoplastic products of fairly good mechanical characteristics which, however, proved to be insufficiently stable electrically.

Meanwhile a thorough study was being made of the electrical and physical characteristics of rubber and particularly of the causes of its electrical instability upon prolonged immersion in water. Our hope that such a study would not only reveal the nature of the defects of rubber but also suggest means for remedying them has been realized to a gratifying degree.

Rubber, as is well known, is also derived from the latex of certain trees, chiefly *Hevea Brasiliensis*. This tree has been

cultivated in large areas on the plantations in the Far East and the product is obtainable commercially in excellently standardized grades. Its principal constituent is a hydrocarbon scarcely distinguishable from that of gutta percha by chemical means, but radically different from it in physical properties, notably in that it has but a slight degree of thermoplasticity and is far more distensible in the cold state. Aside from the hydrocarbon, rubber also contains small amounts of resins, proteins and other impurities, but the aggregate non-hydrocarbon constituents in the better grades are usually less than 10 per cent. in contrast to 50 per cent. or thereabouts for gutta percha and balata.

Rubber is used almost exclusively in industry in a vulcanized form, that is, in combination with a small percentage of sulphur. In this form rubber has also been used to a limited extent for submarine cable insulation, but has long been recognized as lacking sufficient electrical stability for deep sea cables designed to carry a heavy traffic. It is still used to a considerable extent with a fair degree of success for insulation on short cables where the electrical requirements are not severe. In tropical waters it has the advantage over gutta percha of greater resistance to teredo attack and to damage by high temperature.

Some years ago an extended study [1] was made of the causes of the electrical instability of vulcanized rubber, which led to the conclusion that the water soluble impurities are largely responsible. These impurities can be removed comparatively readily and satisfactorily in the process of manufacture, and a submarine insulation of a fair degree of stability is thereby attained.

Even so, vulcanized rubber is very inferior to gutta percha for submarine insulation as the necessary manufacturing operations are more difficult and likely to lead to defects. The removal of mechanical impurities is by no means simple because the raw stock is not plastic enough for thorough straining. The lack of plasticity also interferes with multiple covering of conductors, and the process of heating to bring about vulcanization is liable to result in deformation of the insulating layers. The joining and repairing of core lengths insulated with rubber is also more of a problem than with

gutta percha, which can be so readily remolded in case imperfections appear in the course of the process.

The methods of electrical stabilization of vulcanizable rubber compositions are only partially effective in the absence of vulcanization and it was therefore necessary to extend the study in an effort to secure the desired electrical properties in rubber in the raw state. It might be supposed that mere admixture of raw rubber with gutta hydrocarbon would produce the necessary stability. This is true only to a limited extent. When the proportions of rubber are high enough to meet the mechanical and economic requirements, the electrical stability is impaired.

EFFECT OF PROTEINS ON ELECTRICAL STABILITY OF CRUDE RUBBER IMMERSED IN WATER.

It has been previously shown that crude rubber contains considerable water soluble impurities and that their removal results in a large reduction in water absorption.[1, 2] Rubber so prepared absorbs no more water than good cable gutta percha but in a raw state when immersed in water sooner or later it fails as an insulation often suddenly and completely.

To determine the reason for this electrical instability of crude rubber in water, samples of very pure rubber hydrocarbon completely freed from proteins, resins and other impurities were prepared and tested. It was found that this material not only absorbed very little water but showed practically no change in electrical characteristics as a result of prolonged immersion in water. The impurities natural to rubber, therefore, seem to be responsible for its instability.

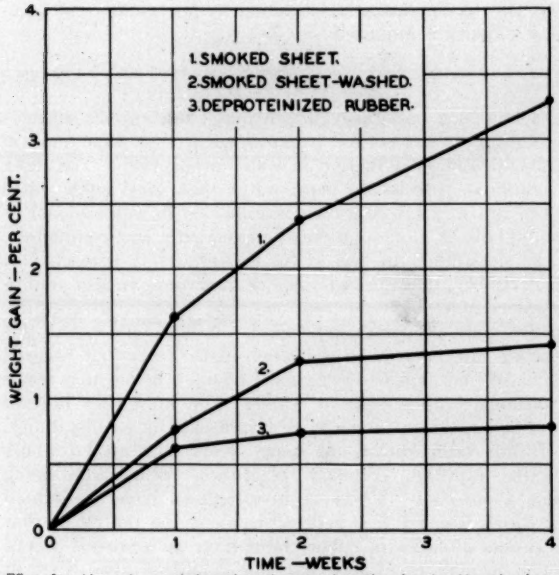
It has been known for many years that crude rubber contains proteins, ordinary plantation rubber containing about 3 per cent. Previous investigators have postulated and shown considerable indirect evidence to the effect that the rubber globules in rubber latex have an adsorbed film of protein around them and that this condition also exists in crude rubber. It is also known that latex serum contains a substantial quantity of protein in solution. The preparation of crude rubber from latex by addition of acid or by processes of evaporation of the water by heat undoubtedly results in the precipitation of considerable quantities of this protein which becomes entrapped between the globules as they coa-

lesce. It is easy then to visualize that in crude rubber there exists a continuous phase of protein or a protein network which, acting like most protein matter, absorbs large quantities of water, resulting in paths through which electrical conduction occurs.

REMOVAL OF NITROGEN CONTAINING BODIES FROM RUBBER.

The problem of developing a suitable commercial method for preparing rubber free from nitrogenous matter offered

FIG. 1.

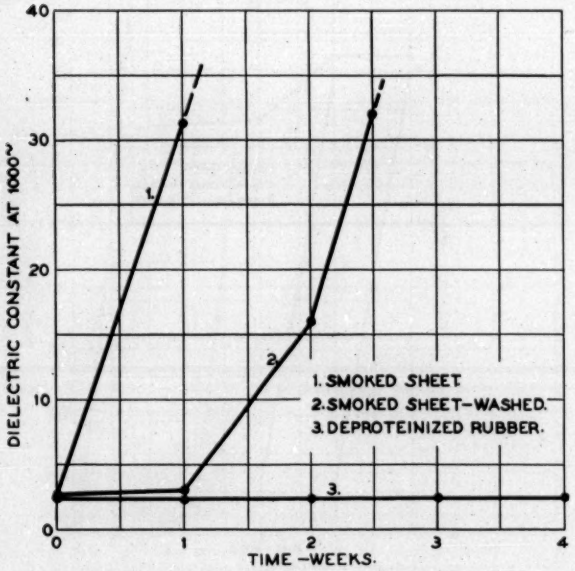


Effect of washing and removal of protein on the water absorption of crude rubber when immersed in 3.5 per cent. NaCl solution at room temperature.

many apparent difficulties. The proteins are colloidal in nature and in the presence of water form gelatinous masses rather than true solutions. On this account they often cannot be removed by simple washing as can be done in the case of gutta percha and balata. It has been known for some time

that proteins can be broken down to water soluble products by boiling with dilute hydrochloric or sulphuric acids. This treatment did not produce satisfactory results in the case of rubber. As a result of many experiments involving a variety of methods, it was found that heating rubber in an autoclave

FIG. 2.

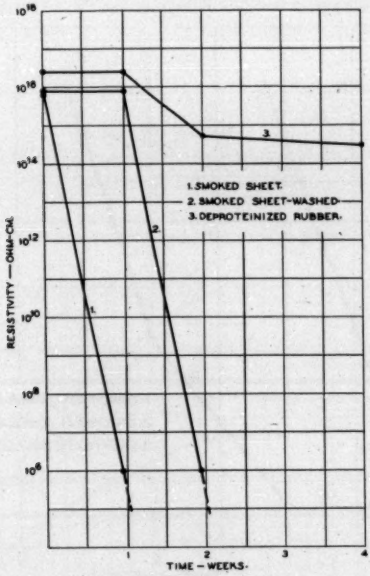


Effect of washing and removal of protein on the dielectric constant of crude rubber when immersed in 3.5 per cent. NaCl solution at room temperature.

at an elevated temperature in the presence of water alone brought about fairly rapidly the desired hydrolysis of the rubber proteins, converting them to water soluble materials. As a result of subsequent washing, it was found that the nitrogenous bodies had been almost completely eliminated without deleterious effect on the rubber hydrocarbon.

Rubber either in the form of sheets immersed in water or as an aqueous rubber dispersion such as latex can be employed in the process. The treatment of latex, however, results in a more rapid hydrolysis of the proteins. Considerable latitude

FIG. 3.

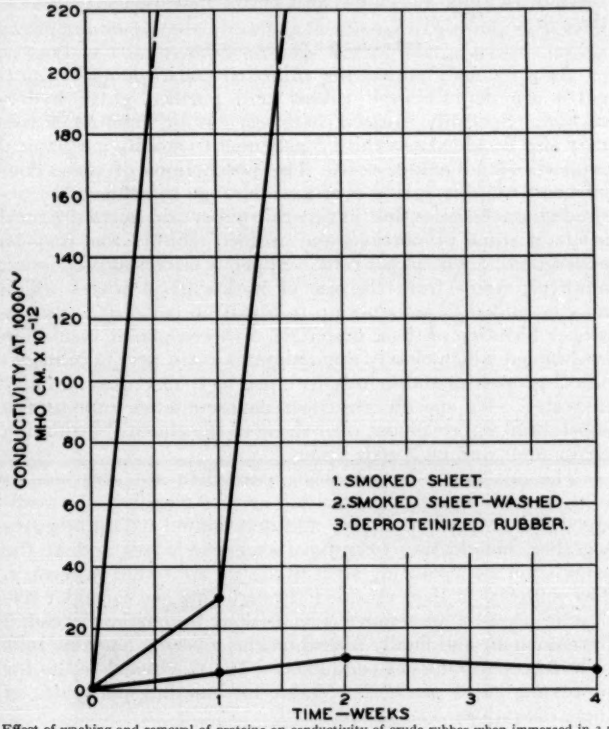


Effect of washing and removal of proteins on resistivity of crude rubber when immersed in 3·5 per cent. NaCl solution at room temperature.

exists in the choice of conditions, but the following example will suffice to describe one method of carrying out the process: ammonia preserved latex is diluted 1 to 5 with pure water. The latex is then heated in an autoclave for approximately ten hours at 150° C. After cooling it is coagulated with acetic acid and thoroughly washed. As a result of this treatment the nitrogen content of the rubber is found to be less

than 0.10 per cent. which is about one fourth that of ordinary plantation crude rubber. Figures 1, 2, 3 and 4 illustrate the

FIG. 4.



Effect of washing and removal of proteins on conductivity of crude rubber when immersed in 3.5 per cent. NaCl solution at room temperature.

relative water absorption and electrical stability of deproteinized rubber as compared with the ordinary crude product. Vulcanized deproteinized rubber was also found to be somewhat superior to ordinary vulcanized crude rubber as regards its electrical stability in water.

In addition to the superior electrical stability of deproteinized rubber, it was found to be more readily plasticized and mixed with gutta than is the case with crude rubber thereby yielding a product with better thermoplastic properties.

PREPARATION OF PARAGUTTA.

As previously stated the principal constituents of paragutta are deproteinized rubber and purified gutta hydrocarbon. Specially treated hydrocarbon or montan waxes may also be added as a third constituent to modify mechanical properties and reduce cost. The proportions of these constituents may be varied over a wide range to achieve the desired characteristics, but in general rubber and gutta are used in about equal proportions and purified montan wax may be added up to about 40 per cent. Superior electrical properties, however, result from the use of hydrocarbon waxes which may be added in amounts up to about 20 per cent. By the proper blending of these materials, a thermoplastic insulation is obtained which closely approximates gutta percha in mechanical properties and is fully its equal as to electrical stability in water. Its specific electrical characteristics represent a substantial improvement over those of the classical insulating compounds and its cost is lower.

The final steps in processing paragutta are very similar to those used for gutta percha and involve blending and washing the deproteinized rubber and deresinated balata or gutta together, masticating to remove excessive water and at the same time incorporating such waxes as are found necessary. The material is then strained through fine sieves under hydraulic pressure to remove adventitious impurities, kneaded to remove air and finally placed on the covering machine rolls to be forced around the conductor. The machinery in use for processing gutta percha is suitable for handling paragutta in these operations.

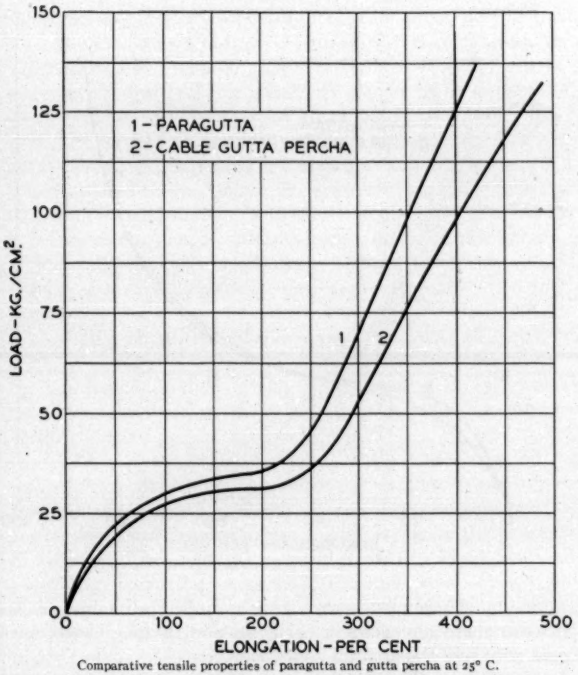
COMPARATIVE PROPERTIES OF PARAGUTTA AND GUTTA PERCHA.

Tensile Properties.—Although submarine insulation is not subjected to tensile deformation in practice, tensile properties indicate to some degree the relative mechanical suitability of a given material for the purpose. Figure 5 shows the stress strain characteristics of paragutta and gutta percha submarine

cable insulation. These results show that paragutta has tensile properties equal to cable gutta percha although its gutta content is substantially lower.

Compression Properties.—The insulated submarine cable conductor commonly known as the core is frequently subjected to uneven compression stresses during manufacture, laying

FIG. 5.

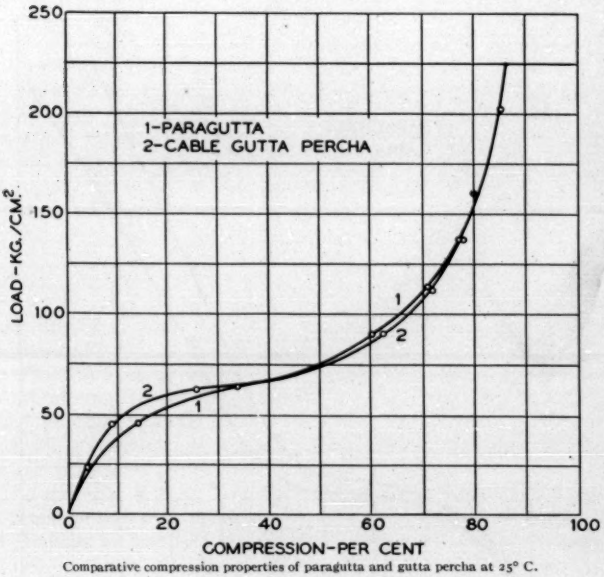


Comparative tensile properties of paragutta and gutta percha at 25° C.

and repairing. The insulation must, therefore, be capable of withstanding these stresses without appreciable deformation. To determine the relative merits of paragutta and gutta per-

cha in this respect their comparative stress strain characteristics under compression have been measured using a special compression machine[3] and are shown in Fig. 6. In this test a steel rod 1.6 cm. in diameter was forced endwise into a sheet of the material .375 cm. in thickness at a rate of about 4 cm. per minute while simultaneously recording the deformation

FIG. 6.



Comparative compression properties of paragutta and gutta percha at 25° C.

and load. These results show that very little difference exists between these materials in this test, and factory handling of cores confirms the general conclusion.

Flexibility.—The flexibility of submarine cable insulation is important because the core is subjected to considerable flexing during manufacture, laying and repairing and possibly at times during use especially where tidal currents may cause

movement in the cable. Paragutta and gutta percha cores have been subjected to slow and continuous flexing at 0° and 25° C. for long periods and it was found that both materials will withstand millions of repeated flexures at small amplitudes without failure. When the amplitude of flexure was increased to strain the conductor slightly beyond its elastic limit, the conductor always failed in advance of the insulation.

Plasticity Tests.—Laboratory tests were made to determine the relative plasticity of paragutta and gutta percha using both the Williams [4] and the Marzetti [5] type of plastometers. These tests are valuable guides but the final judgment of a material as regards thermoplasticity was made by determining its workability on commercial gutta percha insulating machines. Paragutta is somewhat more resistant to flow than gutta percha at temperatures ranging from about 40° to 70° C. When applied to the conductor, however, its greater resistance to flow at elevated temperatures can be taken as an advantage as it lessens the danger of faults occurring if the core should be accidentally exposed to elevated temperatures or to conditions which might exist in connection with cable used in the tropics.

Figure 7 shows the relative plasticities of cable gutta percha and paragutta at several temperatures as determined by the Williams [3] method which can be taken to indicate the relative plasticities of these materials at working temperatures.

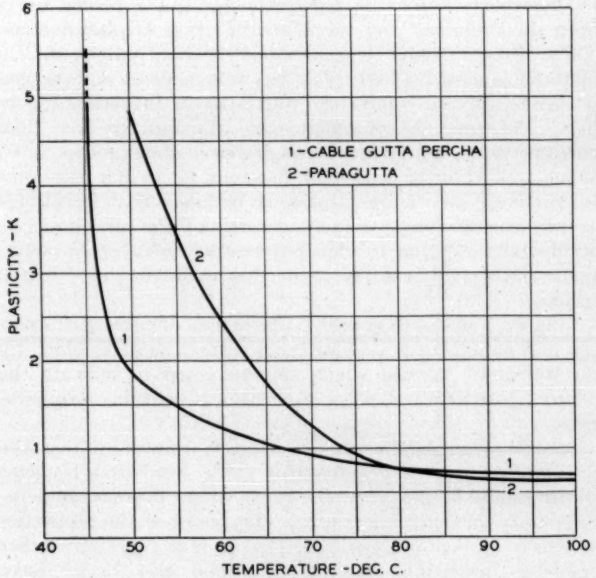
Brittle Temperature.—It is extremely important that the temperature at which submarine cable insulation becomes brittle should be far below the range of sea bottom temperatures to be encountered in use. This is one of the properties in which rubber and gutta percha greatly excel any other available insulating material. Kohman and Peek [6] have described an apparatus for accurately determining this temperature. The brittle temperature of paragutta is somewhat lower than cable gutta percha as can be seen from the results in Table II which give the range of brittle temperature values found for different samples of several materials.

TABLE II.

Brittle Temperature of Paragutta and Other Insulating Materials.

Material.	Brittle Temperature °C.
Gutta Percha (Cable Insulation)	−23 to −36
Paragutta	−45 to −61
Balata (Washed)	−44 to −52
Balata (Washed and Deresinated)	−62 to −67
Crude Rubber	−57 to −58
Vulcanized Rubber (Soft)	−53 to −58

FIG. 7.



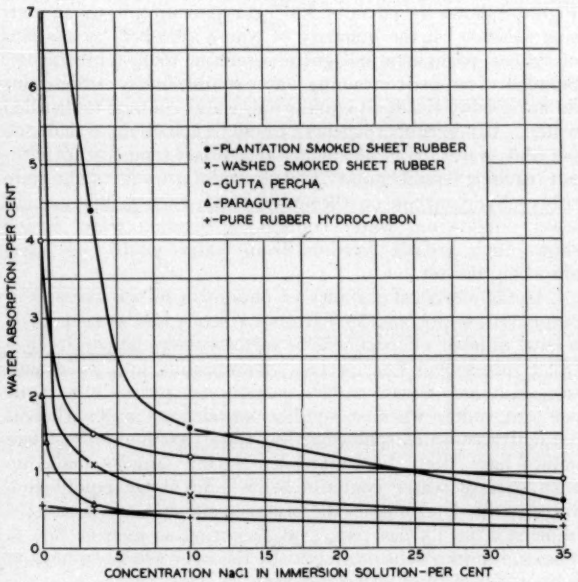
Effect of temperature on the plasticity of cable gutta percha and paragutta.

WATER ABSORPTION—ELECTRICAL STABILITY.

The amount of water absorbed by rubber and gutta percha when immersed in water is the result of a complicated mechanism. The quantity and nature of water soluble or water absorbing impurities in the rubber or gutta percha and the salt concentration of the water in which the samples are im-

mersed are controlling factors. The enormous increase in the quantity of water absorbed by ordinary rubber when immersed in distilled water as compared with its absorption in salt solutions has been explained on the basis of osmotic theory.[1] In accordance with this theory rubber acts as a semi-permeable membrane. Water soluble crystalloids or hydrophillic col-

FIG. 8.



Relation of water absorption to salt concentration in immersion solution.

loids (proteins) attract the water which enters the rubber by diffusion. When immersed in distilled water these impurities tend to reach infinite dilution with water, being opposed in this by the resistance of the rubber itself to swelling. In salt solutions the amount of water absorbed is finite and depends on the equalization of osmotic pressures of the internal and ex-

ternal solutions. The change in water absorption of pure rubber hydrocarbon with the salt concentration of the external solution is small over the whole range which indicates that the water enters by a process of solution. This has also been found to be the case for gutta hydrocarbon and is more or less true for paragutta and gutta percha. The water absorption in distilled water can therefore be taken as a measure of the freedom from water soluble or water absorbing impurities. Figure 8 shows the effect of NaCl concentration in the immersion solution on the quantity of water absorbed by samples of rubber, paragutta and gutta percha at room temperature. Samples of rubber containing water soluble matter or proteins do not readily reach an equilibrium water content in distilled water. Crude rubber has been found to absorb more than 100 per cent. water in distilled water at ordinary temperature without reaching equilibrium.[1] Gutta percha, paragutta and pure rubber hydrocarbon on the other hand reach a definite and lower equilibrium water content in distilled water which shows their greater freedom from water soluble or water absorbing matter.

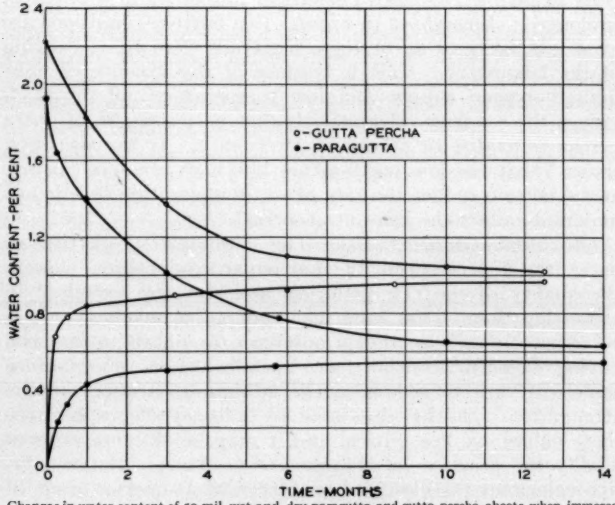
As the electrical stability of paragutta in sea water is of paramount importance an exhaustive study has been made on a large number of specimens as regards their changes in electrical values over long periods of immersion in 3.5 per cent. salt solution. Gutta percha insulation contains about one per cent. water when at equilibrium with sea water whereas paragutta contains somewhat less than this amount. These values have been determined by testing samples made up with various water contents below and above equilibrium values and determining the water content after prolonged immersion in 3.5 per cent. NaCl solution as seen in Fig. 9. The equilibrium value is practically the same when equilibrium is approached from either direction.

The overall quantity of water absorbed, however, cannot be used as a final criterion by which to judge insulation for it has been previously shown (Figs. 1 to 4) that washed crude rubber completely fails as an insulator after absorbing less than one per cent. water. The mode of distribution of water absorbing impurities in an insulating material has been found to be of utmost importance as regards the magnitude of the

effect of moisture in various insulating materials. Examples where large effects on insulating properties are caused as a result of moisture absorption by localized impurities are found in the above case of proteins in crude rubber, water soluble salts associated with fillers in vulcanized rubber [1] and hygroscopic salts on the surfaces of textile fibres.[7]

On the other hand, the electrical properties of paragutta or gutta percha are not impaired when several times their

FIG. 9.



Changes in water content of 50 mil. wet and dry paragutta and gutta percha sheets when immersed in 3.5 per cent. NaCl at room temperature.

equilibrium water content is incorporated with them. Gutta percha, however, does show an increase in capacitance of about 10 per cent. as a result of water absorbed by a completely dried specimen, but as it is always the practice to apply it to the conductor in a wet condition this change is not of practical significance. The electrical properties of paragutta on the other hand show practically no changes as a result of moisture absorption by a dry sample. These facts are taken to be the

best evidence of the electrical stability of paragutta in contact with water.

Hundreds of specimens of paragutta and gutta percha have been studied as regards changes taking place in electrical characteristics after long periods of continuous immersion in 3.5 per cent. salt solution. These tests, some of which have been for periods of three to five years, show that paragutta is fully equal to gutta percha as regards its stability. When properly prepared both of these materials show practically negligible changes in electrical properties as a result of prolonged submergence in water. Sea bottom conditions are even less likely to affect these materials than those existing in the laboratory. This is because of the absence of light limited oxygen supply and low temperature, all of which reduce the tendency of materials such as paragutta or gutta percha to oxidize or otherwise deteriorate. It has also been shown [2] that the low temperature and high pressure existing at sea bottom reduce the rate of water absorption but do not materially affect the amount absorbed.

Electrical Characteristics.—The electrical properties of paragutta depend upon the particular composition chosen, the quality of the raw materials and the care exercised in processing them. For long telephone cable insulation, it is necessary to exercise the utmost care to obtain a material having dielectric constant and specific conductance values sufficiently low to reduce to the minimum its effect on the attenuation. On the other hand for ordinary telegraph cables these values are less critical and it may be advantageous to modify the practice for purposes of economy. Representative values for the electrical properties of a superior grade of paragutta and typical cable gutta percha under sea bottom conditions are given in Table III. It will be seen in this table that paragutta has a 20 per cent. lower dielectric constant and a specific conductance one-thirtieth that of ordinary cable gutta percha under sea bottom conditions. The insulation resistance and dielectric strength of the two materials are practically the same.

TABLE III.

Comparative Electrical Properties of Paragutta and Cable Gutta Percha at Sea Bottom Conditions.

	Specific Inductive Capacity 2° C., 400 Atm., 2000 Cycles.	Effective A-C Conductivity 2° C., 400 Atm., 2000 Cycles Unit = 10^{-12} mho. cm.
Cable Gutta Percha	3.3	90
Paragutta	2.6	3

SUMMARY.

Gutta percha and balata have proven eminently suitable for the insulation of long deep sea telegraph cables, but their dielectric losses are too high to meet the requirements of submarine telephone cables designed to operate over long distances or of shorter cables employing carrier currents.

This paper describes a new material called paragutta which has been developed to meet the present needs. It consists essentially of the purified hydrocarbons of balata (or gutta percha) and of rubber together with minor quantities of waxes to modify the mechanical characteristics. The purification of rubber particularly with respect to proteins is necessary to effect electrical stability in water. A commercially usable method of deproteinizing rubber is described.

Evidence is furnished that paragutta has all of the desirable thermoplastic and mechanical properties of gutta percha while possessing such superior insulation characteristics as to make it suitable for use on long cables designed for transoceanic telephony. Its use is also advantageous on shorter deep sea cables designed for carrier telephony as well as for ocean telegraphs.

ACKNOWLEDGMENT.

The author wishes to acknowledge his indebtedness to Mr. R. R. Williams for counsel and assistance during the prosecution of the work and writing of the paper.

LITERATURE CITED.

1. WILLIAMS AND KEMP. J. FRANKLIN INST., **203**, 35 (1927).
2. LOWRY AND KOHMAN. *J. Phys. Chem.*, **31**, 23 (1927).
3. HIPPENSTEEL. *Bell Laboratories Record*, **5**, 153 (1928).
4. WILLIAMS. *J. I. E. C.*, **16**, 262 (1924).
5. MARZETTI. *Giorn. Chim. Ind. Applicata*, **5**, 342 (1923).
6. KOHMAN AND PEEK. *J. I. E. C.*, **20**, 8 (1928).
7. WILLIAMS AND MURPHY. *Bell System Tech. J.*, **8**, 225 (1929).

The Disembarkation of Passengers in Emergency at Sea.—E. F. SPANNER. (North-east Coast Institution of Engineers and Shipbuilders.) The modern passenger ship may be compared to a huge hotel subdivided into floors and sections by horizontal and vertical divisions. Just as there is necessity for properly planning emergency exits from a hotel in case of fire, so there is need properly to plan emergency exits from a ship in case of threatened disaster.

The problem in the case of a ship is complicated by the facts: (1) That in emergency, the vessel may not be floating upright. (2) That she may be rolling and pitching. (3) That the passengers have to be evacuated or disembarked from a pre-determined disembarkation deck. (4) That the passengers have to be disembarked into boats swinging from points high in the ship, and partaking of the movement of the ship.

The Author approaches the study of the problem of disembarking by considering first the problem of providing the necessary boats and boat-handling gear, passing from that to the task of assembling the passengers for the purpose of rapid and efficient disembarkation. Particular attention is drawn to the necessity of ensuring that all passengers have, at least, adequate facilities to reach the disembarkation deck.

In conclusion the author points the way toward betterment of existing practice, incidentally setting out certain new proposals in connection with the disposition and handling of the boats. R.

Catalytic Oxidation of Mercurous Nitrate Solutions by Cupric Ion. By FREDERICK G. GERMUTH. (*American Journal of Pharmacy*, May 1930.) Data are furnished showing the effect of catalytic action upon varying concentrations of mercurous nitrate solutions by cupric ion, after certain intervals of time.

The presence of the bivalent ion in mercurous solutions acidified with nitric acid causes a pronounced acceleration of the reaction occurring between atmospheric oxygen and mercurous ion, whereby the latter undergoes conversion to the mercuric state. This change is further favored by dilution.

The anion present in the cupric salt determines, apparently, the rapidity with which catalysis is induced. It is possible that differences in solubility, dissociation, and indirectly related phenomena may account wholly or in part for this observation.

Increased amounts of cupric nitrate in excess of that proportion utilized as catalyst in this study do not materially affect the rate at which oxidation occurs.

A PORTABLE SPARK CHRONOGRAPH FOR USE ON EITHER DIRECT OR ALTERNATING CURRENT.

BY

C. N. HICKMAN, Ph.D.

A chronograph, having a relatively high degree of precision, is a very useful piece of apparatus in many laboratories.

The simplicity of operation and the speed with which measurements may be taken, makes the spark chronograph very desirable.

It is frequently desirable to make measurements outside of the laboratory. In this case the current available is usually 110 volts A.C.

The chronograph, described in this paper, was designed so that it could be used on either direct or alternating current, and on any voltage from 50 to 250. It may readily be carried from place to place and quickly adjusted to the current available.

The instrument, as constructed, is capable of recording six events. The number of events could be increased without materially increasing the cost of construction.

Figure 1 shows a cross section of the chronograph. *M* is a series wound motor, provided with ball bearings. The shaft (*S*) carries the rotor (*R*) and governor frame (*F*). The latter (*F*) is provided with a governor arm (*g*), hinged at *h*. The weight (*w*) is attached to the arm (*g*) and is counterbalanced by the spring (*sp*). The arm (*g*) carries a contact member (*s*), which bears against a contact screw mounted on the bracket (*Br*). A lock nut (*n*) holds the lower contact member in any desired position. These contact members are shunted across a resistance which is in series with the motor windings.

When the speed of the motor is low, the contact at (*s*) is closed and the full voltage is applied to the motor. When the speed of the motor reaches a value sufficient for the centrifugal force of the weight (*w*) to overcome the tension of the spring (*sp*), the contact (*s*) is opened. This places a

resistance in series with the motor and thus lowers the voltage so that the motor speed decreases. The making and breaking of the contact (*s*) results in a relatively constant motor speed.

The principle of this governor is the same as that used in the "Aberdeen Chronograph" developed by Major Loomis and Dr. Klopsteg during the World War, for measuring the velocity of projectiles.

FIG. 1.

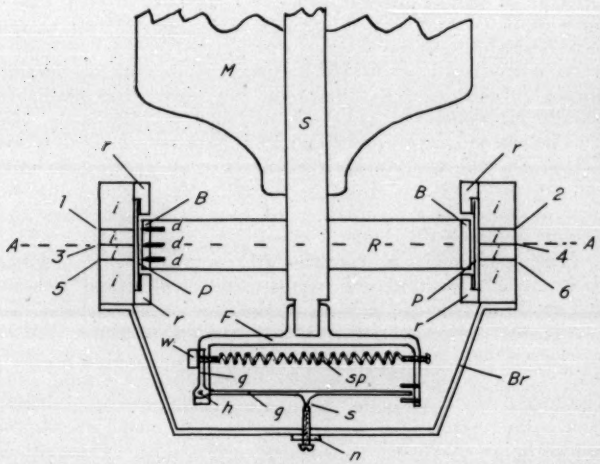


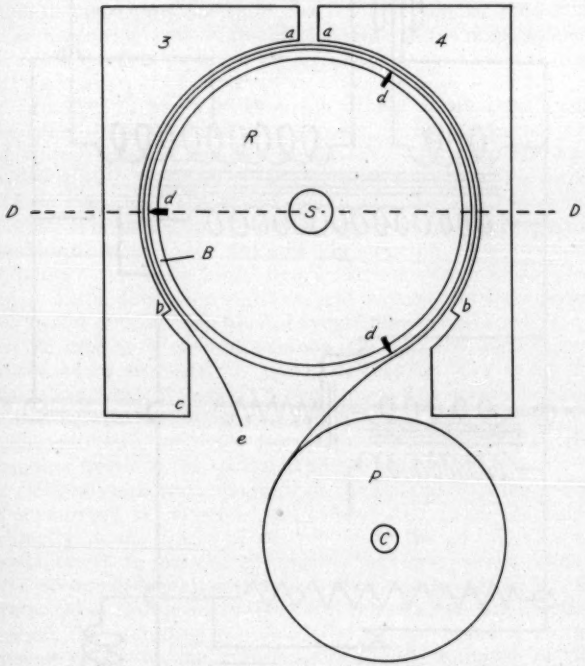
Figure 2 shows a section of the chronograph at *AA* of Fig. 1. *S* is the motor shaft, which carries the metal rotor (*R*). The circumference of the rotor (*R*) is covered with a ring of bakelite (*B*). The metal pins (*d*) are located 120 degrees apart. These pins are set in the metal rotor (*R*) and project through the bakelite ring (*B*) so that their points are flush with the surface.

A roll of chronograph paper (*P*) is supported on a core (*C*). The paper passes around the rotor. After a record has been taken, the end of the paper (*e*) is used to pull a new

section into position. The paper which has been pulled out may be torn off on the corner (*c*) if desired.

The segments 3 and 4 are made of metal and are connected to the terminals of the secondary spark coils. The spark

FIG. 2.

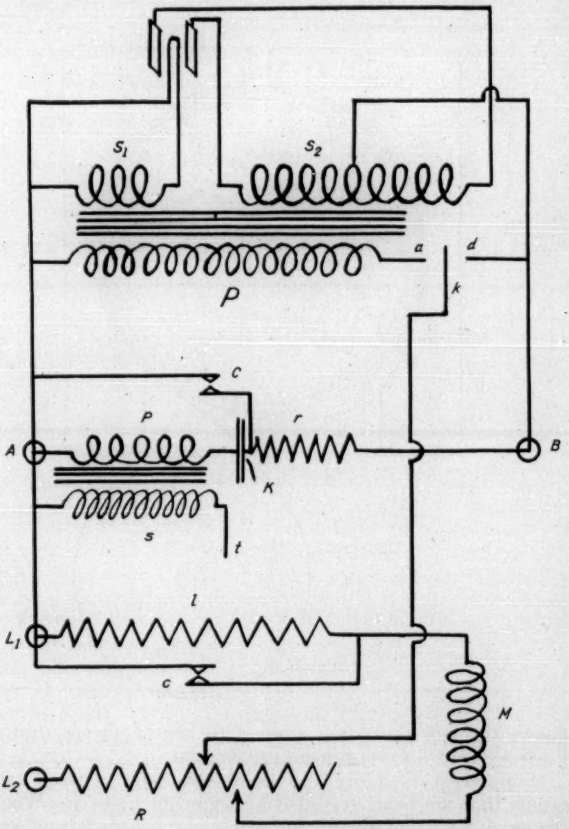


passes through the paper from *d* to one of the segments. Each segment (*ab*) represents 120 degrees.

Suppose it is desired to know the time between two events that are being recorded by segments 3 and 4. When the spark passes from *d* to segment 3, a record is left on the

paper. If both events occurred simultaneously, another spark would pass from another (*d*) to segment 4. The distance between these records on the paper would be equal to

FIG. 3.



one-third the circumference of the rotor and would be labeled zero on the measuring scale. If the second event did not occur simultaneously, but, say, .003 second later, the rotor would have traveled a certain distance in this time. The distance between the records on the paper would then be equal to either the distance moved over by the pin (*d*), or to this distance plus one-third the circumference of the rotor. The measuring scale is therefore double and is made so that it reads the time between the events directly in thousandths of a second.

In Fig. 1, which is a section of Fig. 2, at *DD*, it will be seen that there are six segments. These segments are mounted between bakelite blocks (*i*). There are also two bakelite rings (*r*) which guide the paper (*P*) around the rotor. These rings hold the paper about .007 inch away from the rotor. The space between these rings and the bakelite (*i*) is sufficient to take the thickest chronograph paper without binding. As shown in the figure, there are three sets of pins (*d*). By adding more segments and a corresponding number of sets of pins, any number of events may be recorded.

In case it is desired to know the time between events recorded by segments (1, 3 or 5) or between (2, 4 or 6), the measuring scale is used in the same manner as in the Aberdeen Chronograph, i.e., only one section of the scale is used and measurements are made from either end according to the manner in which the sparks happen to record.

Figure 3 shows a diagram of the electrical wiring. The line current is connected to L_1 and L_2. L_1 is connected directly to the frame of the motor, to the governoring resistance (*l*), to one contact member of the governor contact (*c*), to one terminal of each secondary spark coil (*s*), to one terminal of each primary spark coil (*p*), to a common terminal of all the recording contacts (*C*), to one terminal of the primary (*P*) of the transformer, to one terminal of the secondary (S_1) of the transformer, and to the filament of a 180 rectifying tube.

The other side of the line is connected to a variable resistance (*R*). The motor windings (*M*) are in series with the resistance (*l*) and a part of the resistance (*R*).

The other terminal (*t*) of the secondary spark coil (*s*) is connected to one of the segments of the chronograph.

A 4 microfarad condenser (K) is in series with the primary (p) of the spark coil and a 1000 ohm resistance (r).

The middle of the secondary coil (S_2) of the transformer is connected to B, which is a common terminal for all of the circuits (p K r).

The knife switch (k) is connected to (a) for alternating current and to (d) for direct current.

In any case there is a direct current of about 100 volts across the condenser (K).

When the contact (C) is closed, the condenser discharges through (p) and induces a current in the secondary (s). A spark passes from (t), which is connected to one of the segments (1, 2, 3, 4, 5 or 6), to the frame of the motor through one of the pins (d) in the rotor.

A 25-watt lamp is used for resistance (l), so that by its flicker, one is assured that the governor is functioning properly.

The value of (R) is easily changed to suit the voltage of the current available.

The spark coils used in the chronograph are Ford, Model T ignition coils.

Chronograph paper such as was developed for the Aberdeen Chronograph may be used. Another very satisfactory paper is described in the October, 1929, issue of the magazine published by the Acoustical Society of America. The subject of the article that describes this paper is: "A Spark Chronograph Developed for Measuring Intensity of Percussion Instrument Tones."

The width of the paper is one and one-sixteenth inches, which is the same as that used in the Aberdeen Chronograph.

Excellent records are obtained on the Aberdeen Chronograph paper with voltages as low as 80 volts. Readable records are obtained with voltages as low as 50. By using the special paper mentioned above, voltages as low as 20 may be used.

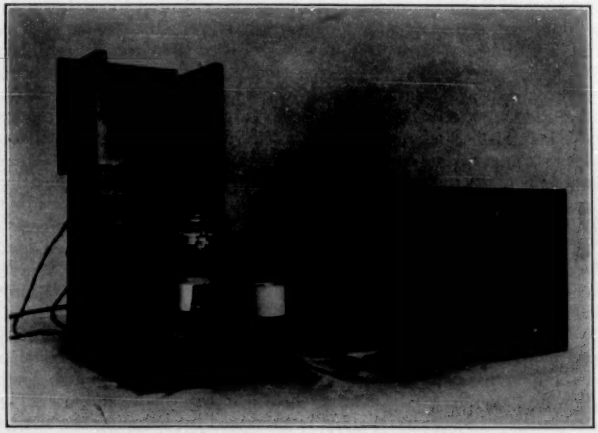
The speed of the motor is 2000 R.P.M. One revolution takes .030 second. One-third revolution, which is the length of a segment represents .010 second.

The photograph shows the complete chronograph equipment. The rectifying system is in the box containing the

motor. The spark coils, condensers and resistances are in the other box.

Figure 4 shows that the apparatus is not bulky. It is relatively light in weight.

Fig. 4.



Measurements made with the chronograph, using 110 A.C. have shown that an accuracy of better than .1 per cent. may be obtained.

The Application of X-Rays to the Study of Metals. J. B. Friauf. (*Rev. Sci. Instruments*, July, 1930). X-rays are used in three different ways in studying metals,—a. radiographic examination for flaws, etc.; b. determination of chemical composition by X-ray spectroscopy, and c. examination of crystalline structure. Only the first will be considered here.

In radiographic examination for cracks, flaws and cavities the X-rays cast a shadow of the material under examination either upon a fluorescent screen or on a photographic plate. If there be a cavity the rays that pass through it are less absorbed than are adjacent rays and hence produce a stronger effect when they attain the registering surface. The cavity thus is outlined on the shadow. If it is desired to locate its position within the body a second photograph can be taken with the X-ray tube in a different position. Several factors go to determine the size of cavity that can be detected. These are, a. penetrating power of the rays. The larger the absorption coefficient of the wave-length used the smaller the cavity discernible. This has the objection that thereby the time exposure is increased. "X-rays generated by voltages up to 300 kilovolts have been used for the examination of 4 1/2 inches of steel." b. Scattered X-rays. When the primary beam falls on the mass of metal all parts emit scattered X-rays, passing out in all directions. These dim the shadows cast by cavities and make them harder to detect. A Bucky grid helps to reduce the amount of scattered radiation. It consists of metal strips placed between the tube and the specimen so as to allow the rays to pass between them. c. Size of focal spot. The sharpness of the cavity shadow is enhanced by reducing the size of the focal spot from which the rays proceed and by moving it farther away. Both of these are subject to limitations. "A focal distance of some 40 to 50 cm. seems to be the usual practice for the examination of the heavier metals, while as much 62 inches has been used for aluminum." d. photographic technique.

Castings are examined for gas cavities and for other defects. In one case a radiographic examination was made of a 1200 lb. pressure steam line. Weaknesses and flaws in forgings and worked metals come to light under X-ray examination. Its high cost is a drawback.

G. F. S.

THE PHOTOGRAPHY OF WAVES AND VORTICES PRODUCED BY THE DISCHARGE OF AN EXPLOSIVE.[1]

BY

D. B. GAWTHROP,[2] W. C. F. SHEPHERD,[3] AND G. ST. J. PERROTT.[4]

Previous papers [5] by the writers have presented data on the speeds of propagation of the disturbance across a gas gap separating two cartridges of explosive when one cartridge—the "initiator"—is exploded by a detonator, and the second cartridge—the "receiver"—is detonated by influence. It was shown that this disturbance had the characteristics of a discontinuous wave motion of high intensity. Some knowledge of the relative positions at any time of the wave phenomena, the flame and the hot gases of the detonation, is necessary to explain fully the "distance" effects produced by an explosive.

The present paper describes preliminary experiments in which the phenomena sent out by the detonation of a charge of explosive have been photographed by means of the Schlieren method. In this method, the wave phenomena are rendered visible by an optical device. This preliminary research will later be amplified to test the theoretical conclusions on the transfer of detonation across a gas gap to which reference is made above.

PRINCIPLES OF SCHLIEREN PHOTOGRAPHY.

The Schlieren method of photography was developed by Töpler [6] in 1867 and has been employed by Mach [7] and co-

[1] Published by permission of the Director, U. S. Bureau of Mines. (Not subject to copyright.)

[2] Assistant explosives engineer, Pittsburgh Experiment Station, U. S. Bureau of Mines.

[3] Assistant, Safety in Mines Research Board, Great Britain.

[4] Superintendent, Pittsburgh Experiment Station, U. S. Bureau of Mines.

[5] Perrott, G. St. J., and Gawthrop, D. B., JOUR. FRANKLIN INST., 1929, **208**, 643–660, and 1927, **203**, 387–406.

[6] Töpler, A., *Ann. der Phys. u. Chem.*, 1867, **131**, 33–35.

[7] Mach, E., *Ann. der Phys.*, 1876, **159**, 330–331.

workers, Allen,[8] Cranz,[9] Payman [10] and co-workers to photograph such phenomena as atmospheric disturbances produced by high-speed projectiles, heated gases from flames and percussive waves produced by detonating charges of explosive. Payman gives a comprehensive survey of the literature and a description of the method.

Two methods are available for the photography of atmospheric waves and disturbances—the Schlieren method of Töpler and the shadow method of Dvòrak.[11] Of the two, the shadow method is more simple to operate but it is more limited in its application. Both methods depend on the principle of refraction of light at a surface of optical discontinuity.

Shadow effects may be readily observed on a screen when hot gases from a flame are allowed to rise through a divergent beam of light which is illuminating the screen. An arrangement of this type forms the essential part of an apparatus for taking shadow photographs, in which a plate or film replaces the observation screen. The formation of these shadow effects may be explained simply as follows: Imagine that a thin spherical glass shell is placed in a divergent beam of light from a point source. There will be seen on the screen a narrow circular band of shade with an inner contiguous band of brightness on an otherwise evenly illuminated field.[12] The circle of shade corresponds to a true geometric shadow of the glass shell, and the light which is "missing" in the shaded band goes to form the inner bright circular band. It is evident that rays of light which just strike the outside edge of the spherical shell on a plane parallel to that of the screen will be refracted inward, because the refractive index of glass is greater than that of air. These rays form the bright circle and the circle which they originally illuminated will now receive little or no light. All other rays which strike the spherical shell pass through the air inside the shell and it is easy to show that, on

[8] Allen, H. S., *Proc. Royal Phil. Soc.*, Glasgow, 1920, **33**, 71.

[9] Cranz, C., and Bames, E., *Zeit. f. Ang. Chem.*, 1923, **36**, 76–80.

[10] Payman, W., and Robinson, H., The Pressure Wave Sent Out by an Explosive: Part I: Safety in Mines Research Board Paper No. 18, 1926.

[11] Dvòrak, —, *Ann. der Phys.*, 1880, **9**, 502.

[12] Neglecting absorption of light in the glass.

leaving the shell, they will travel along a path which is a continuation of their original path. They are not permanently displaced or refracted, as are the rays which strike the periphery of the shell, and do not pass inside the shell. These same shadow effects would be produced by a shell composed of any medium of which the refractive index is different from that of the surrounding atmosphere. A shell of hot air or of compressed air would produce the shadows. The circle of shade is outside or within the circle of light according as the refractive index of the medium composing the shell is greater or less than that of the surrounding medium. In a photographic shadow apparatus, the plate or film must be relatively near the disturbance, hence the method is not suitable for explosive research in which high pressures are suddenly produced.

In the Schlieren method, light from the source falls on a concave mirror and is reflected as a convergent beam. This beam comes to a focus on the front surface of a lens and there forms an inverted image of the source. This front surface of the lens is half covered by an opaque semi-circular diaphragm. The image of the source formed by the convergent beam falls for the most part on this diaphragm, the remainder falling on the lens and passing through to form on a screen an illuminated circle corresponding to the mirror. The disturbance to be observed is produced in the convergent beam between the mirror and the lens. The screen or image plane of the lens is so placed that the corresponding object plane lies in the principal plane of the disturbance. Imagine that the spherical glass shell is now placed in the beam of light. As in the shadow method, it is only the rays of light which strike the edge of the shell on a circular section in a plane parallel to the screen, which are permanently refracted. All other rays travel ultimately along a continuation of their original path. The rays of light which are refracted are bent inward, and they converge to form a secondary image of the source at a point removed from the opaque diaphragm; as a result, they are diverging again when they reach the diaphragm and some of the rays which originally fell on the diaphragm now fall on the lens, and others which passed through the lens now fall on the diaphragm. By elementary optics, it will be evident

that the rays which are bent into the lens will form a bright semicircular line on the screen and that the rays which are bent out will form a dark semicircular line of shade—the two forming a complete circle on the otherwise evenly illuminated circle of light corresponding to the mirror. The diameter joining the two halves of the circle is parallel to the straight edge of the semicircular diaphragm, and as the lens is focussed on the plane in which the light rays are refracted and from which—to the lens—they appear to come, the circle of light and shade on the screen will be a sharply defined outline of the spherical shell. The substitution of a shell of a medium optically less dense than the surrounding atmosphere would give a similar outline on the screen with this difference—the semicircular lines of shade and bright would now be transposed.

The foregoing is a simple theoretical treatment of the Schlieren method and it will be clear, even for a solid sphere of a transparent medium, that only rays of light which strike the edges are refracted to any extent. The only necessary condition is that there should be a sudden change of optical density at the bounding surface of the sphere. It is in this way that the Schlieren principle makes visible a shock-wave or a body of heated gas; for in both cases sudden optical changes occur at the bounding surface of the phenomena.

Two different types of Schlieren apparatus have been designed on this principle to photograph phenomena invisible to the naked eye, the first to take snapshot photographs, in which case the screen is replaced by a stationary photographic plate; the second to take continuous photographs on a moving film, and in this case a continuous source of light is required.

Snapshot photographs require an exposure in which no appreciable movement of the phenomenon takes place—for explosive phenomena, the exposure must be of the order of 10^{-6} second—and this extremely short exposure must be made when the phenomena to be photographed are in the field of view. These conditions are fulfilled by using a spark discharge as the source of light and by controlling the time that this illuminating spark [13] passes by the phenomena to be photographed. Snapshot photographs of the discharge of an explosive are instructive. They have shown, for instance,

[13] See Safety in Mines Research Board Paper No. 18, cited.

that the shock-wave sent out assumes a spherical form in unconfined space. They do not show, however, the relative positions of shock-wave and flame at the moment the illuminating spark passed, for a time exposure of the flame is super-imposed on the Schlieren record, and the photograph shows the position the flame finally reached.

For the purposes of the present research on explosives, more valuable information was to be obtained from moving film photographs which would show the relative positions occupied at all times by the shock-wave, flame and products from the detonation of the explosive; hence, we have used the Schlieren "wave-speed" camera developed by Payman and Shepherd.[14] We have also the added advantages with this camera that the speeds of propagation of the phenomena can be computed from the photographs, which represent time-distance diagrams.

DESCRIPTION OF THE WAVE-SPEED CAMERA.

The apparatus is shown diagrammatically in Fig. 1. Rays of light from the arc *a* are brought to a focus at *b* and

FIG. 1.

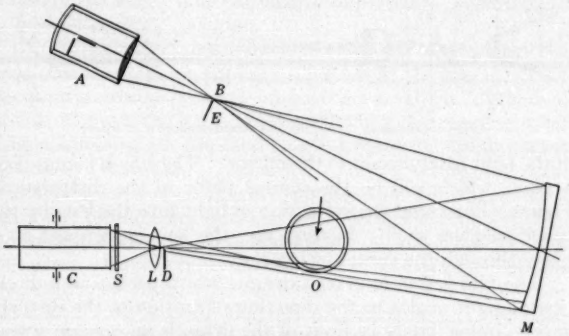


Diagram of the Schlieren photographic apparatus.

then diverge. That part of this diverging beam which falls on the concave reflecting mirror *m* is brought to a focus on the

[14] Payman, W., and Shepherd, W. C. F., "The Pressure Wave Sent Out by an Explosive," Part II, Safety in Mines Research Board Paper No. 29, 1926.

opaque semicircular diaphragm *d* in front of the lens *l* to form an image of *b*. The diaphragm *d* is placed in front of the lens so that its diameter is at right angles to the main axis of propagation of the explosive phenomena. The amount of light that passes through the lens has to be accurately controlled. To give precision to this control, an adjustable screen *e* placed at *b* gives a definite straight edge to the image formed at *d*. This straight edge is parallel to the diameter of the diaphragm *d* and, in the critical position of *e*, the two coincide, in which case the image of *b* falls entirely on the diaphragm *d*. A slight lateral movement of *e* out of the light focus at *b* will now allow a narrow band of light to pass through the lens 1.

The concave mirror *m* is made of stainless steel and is 46 cm. in diameter. It has a radius of curvature of 3.66 meters.

The disturbance to be photographed takes place in the converging beam between the mirror and the lens, sufficiently removed from the mirror to allow an uninterrupted passage

FIG. 2.

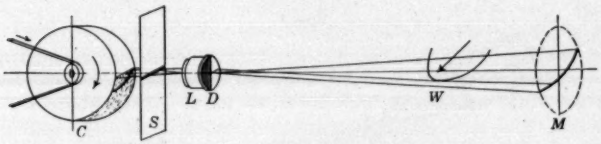


Diagram illustrating the photographic record obtained from a wave.

of the light from the arc to the mirror. The lens is focussed on a plane which will be the central plane of the disturbance. Figure 1 illustrates the refraction of light into the lens by the spherical glass shell. In this case, the lens is focussed on a plane through the center of the shell.

The film is mounted on a drum *c* which rotates in a direction at right angles to the direction of motion of the disturbance. Immediately in front of the drum is an opaque screen *s* in which is cut a narrow stenopaic slit. Figure 2 shows the apparatus from another angle. The screen shields the film from all Schlieren effects caused by the outward spread of the disturbance, and the film is only affected by light which comes from that part of the object plane which corresponds to the

stenopaic slit in the image plane. The position of the screen is adjusted so that the slit corresponds to the main axis of propagation of the disturbance as shown in Fig. 2.

The film drum *c* is one meter in circumference and was driven at a peripheral speed of 60 m.p.s. The revolutions per second were observed direct from a magneto-type tachometer. Exposure of the arc light to the film is made by means of a shutter which in falling freely past the slit in *s*, exposes the film for one revolution of the drum. The shutter consists of a metal plate in which is cut a rectangular hole of the requisite dimensions. An electrical contact arm on the falling shutter is so adjusted to fire the shot that the phenomena to be photographed are in the field of view when the shutter is open.

NORMAL AND REVERSED SCHLIEREN PHOTOGRAPHS.

The diaphragm *d* may be placed to cover either half of the lens. Figure 2 shows how the shock-wave *w*, by bending extra light into the camera, leaves a trace on the rotating film *c*—the dark trace corresponding to the bright produced by the extra light and the general shade to the light which is arranged to fall on the lens. The diaphragm so placed is considered to be in the "normal" position.

If the diaphragm were placed to cover the other half of the lens in this illustration, the light refracted by the passage of the shock-wave would be bent on to the diaphragm instead of passing through the lens. This results in a decreased amount of light affecting the film and this setting of the diaphragm is said to be "reversed." Thus, for a normal setting the trace of a shock-wave on a finished photograph will appear as a white line on a darker background. With a reversed setting the trace of a wave will be a black line. Hence, it will be clear that when a wave is reflected the color of its trace changes, for the Schlieren setting is normal for one direction of trace of the wave and reversed for the opposite direction. The characteristics of phenomena photographed with normal and reversed Schlieren settings as they will appear on a finished print are given in Table 1.

TABLE I.

Characteristics of the Phenomena Photographed by the Schlieren Method.

Phenomenon.	Appearance in Schlieren Photograph. Normal Setting of Diaphragm.	Reversed Setting of Diaphragm.
Incident wave	Bright line	Dark line
Reflected wave	Dark line	Bright line
Boundary of moving gases:		
Density less than air	Dark line	Bright line
Density greater than air	Bright line	Dark line
Projected gas vortex:		
Density less than air	Dark line followed by bright line	Bright line followed by dark line
Density greater than air	Bright line followed by dark line	Dark line followed by bright line
Solid particles too small to be photographed	Fine bright line followed by fine dark line	Fine dark line followed by fine bright line
Mists of fine particles (smoke)	Dark area	Dark area
Flame directly photographed	Bright	Bright
Flame not sufficiently actinic for direct photography	Dark line	Bright line

CHARGES OF EXPLOSIVE FIRED IN AIR.

In previous papers,[5] the authors have reported on experiments carried out to explain the transfer of detonation from one cartridge of explosive to another when the two cartridges are separated by a gas gap. Two methods were adopted, the first in which the two cartridges were freely suspended, end on, in the air; the second, in which the two cartridges were contained in a paper cylinder of the same diameter as the cartridge of explosive. It was found that the paper cylinder had a large effect in lengthening the gap over which detonation was transmitted.

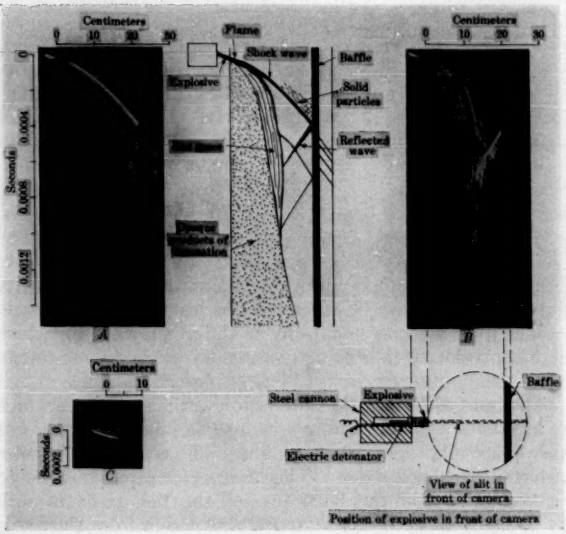
The preliminary research to be described in the present paper has sought to interpret and identify the various phenomena sent out by the detonation of cartridges of explosive fired in the open, and at one end of a cylinder. It is hoped that this will be valuable for later research in which the second cartridge will be added and further gap-test experiments carried out using Schlieren photography as the method of analysis.

The photographs will now be described:

Figure 3.—A charge of 40 per cent. straight dynamite was packed in a cellophane wrapper to form a cartridge 8 mm. in

diameter and 7.6 cm. in length. The approximate weight of the charge was 6.0 grams. The electric detonator employed to detonate the explosive was confined in a small steel cannon and one end of the cartridge was placed just inside the bore of the cannon. The detonator was confined to prevent flying pieces of copper from striking the surface of the steel mirror. A wooden baffle 15.0 cm. wide was placed at a distance of 30.5 cm. from the end of the explosive charge. The disposition

FIG. 3.



of the cartridge, baffle, etc., relative to the field of view is shown in the figure.

Two Schlieren photographs of the discharge of the explosive are shown together with an explanatory diagram. One photograph, 3*A*, was taken with a normal setting, and the other, 3*B*, with a reversed setting. A direct moving film photograph, 3*C*, of the flame from the explosive is included.

Flame from the detonating explosive cartridge photographs directly. Particularly in photograph *B*, the Schlieren effect of the hot gases from the explosive is sufficient to cause the flame propagated from the outer end of the cartridge of explosive to be indistinct. A direct moving film photograph of the flame without any Schlieren photography shows the length of the flame to be about 5.0 cm. Initially, there is no distinguishable time interval between flame and the shock-wave.

The spherical shock-wave advances and is reflected from the baffle which is indicated in the explanatory diagram at the side of Fig. 4. The wave also passes around the baffle and in doing so its forward movement is arrested for a time depending upon the width of the baffle. Other waves following may be due to reflections from the small steel cannon, continued reaction of the explosive ingredients, or projection of small particles. Projection of flying particles which did not strike the baffle is evident from the number of fine bicolored lines photographed at the side of the baffle away from the explosive. Small solid particles break through the shock-wave at a distance of 8.0 cm. and greater from the end of the explosive. The average velocity of the shock-wave over the first 8.0 cm. of propagation is 1100 meters per second, hence, the velocity of the particles must be greater than this. The particles do not lose their velocity as rapidly as the shock-wave for which the mean velocity falls to 640 meters per second over 20-cm. travel.

The hot gases from the detonation of the explosives which are less dense than air and in which some mixing with air has taken place, photograph as a number of lines opposite in color to that of the shock-wave. The effect of air pressure between the hot gases and the baffle causes the hot gases to stop advancing, followed by a temporary recession from the baffle which may be due to the impact of the reflected wave. Opaque products of detonation following the hot gases blot out any further photographic record.

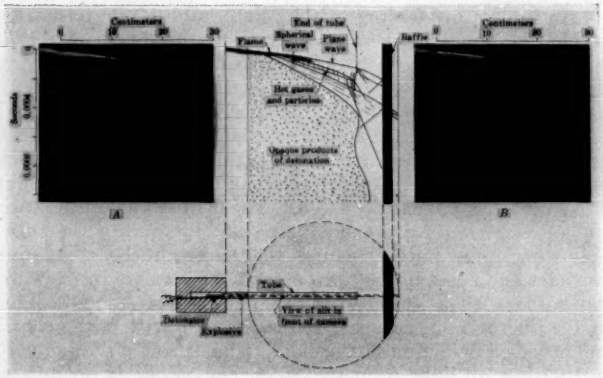
CHARGES OF EXPLOSIVE FIRED IN CELLOPHANE TUBES.

Previous mention has been made of the effect of a paper tube on the length of the gas gap over which detonation is

transmitted. The use of cellophane instead of paper as the material of the tube enables Schlieren photographs to be taken of the phenomena as they travel along within the tube.

Figure 4.—This experiment was carried out to ascertain the nature of the phenomena which proceed through the tube and outside of it from the initiating cartridge. There was no "receiving" cartridge.

FIG. 4.



A charge of 40 per cent. straight dynamite 7.7 cm. in length was packed in one end of a square cellpohane tube 8 mm. in width and 29.5 cm. in length. The arrangement of the charge of explosive within the cellophane tube and the various dimensions are given in the figure. A wooden baffle 23 cm. in width was placed at a distance of from 5.5 to 6.3 cm. from the end of the cellophane tube and facing the charge of explosive. The two photographs in Fig. 4 were taken with normal and reversed Schlieren settings. The distance scale is double that of Fig. 3; the time scale is unchanged.

The flame of the explosive registers on the film by reason of its own actinic qualities and travels a distance of 12.5 cm. from the end of the charge. The luminous effect or flame resulting from the discharge of an explosive at one end of the tube has been described by the authors in a previous paper.[5]

A plane shock-wave is initiated inside the cellophane tube. For the first part of its travel it is contiguous with the flame and this wave has an average velocity of 2040 meters per second throughout the length of the tube. The shock-wave is shown on the photographs as a thin line extending from the flame. When the wave emerges from the mouth of the cellophane tube is begins to expand spherically with a consequent decreasing velocity. This spherical wave is reflected at the surface of the baffle.

An interesting feature of the photograph is the formation of what is thought to be a vortex ring by the shock-wave as it emerges from the mouth of the tube. Further evidence of this vortex formation will be advanced later. The vortex ring travels more slowly than the shock-wave.

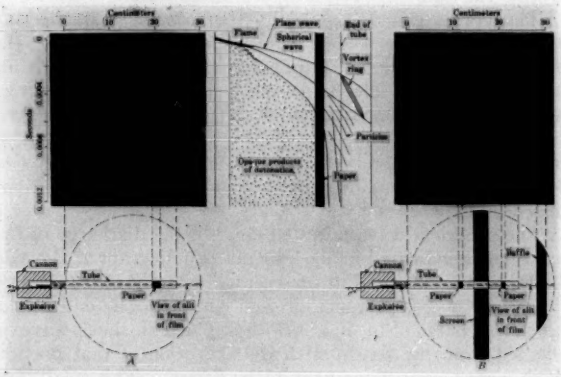
The cellophane tube appears to buckle and possibly break up after the plane-wave passes through it. Dark lines, which may be caused by wrinkles or pieces of the tube, show a slight movement away from the explosive near the flame in the tube and movement toward the explosive at the end of the tube. Following immediately behind the flame in the tube, fine particles and gases in the form of a smoke are shot out toward the baffle with a velocity of 850 meters per second. This smoke photographs as a narrow black region with both normal and reversed Schlieren settings due to its action in temporarily cutting out all light from the arc. The smoke is opaque to light and leaves a sooty deposit on the baffle.

In addition to the plane shock-wave sent along the tube, the detonation of the cartridge results in the formation of spherical shock-waves, which spread out with rapidly decreasing velocities. The trace of the first and main wave is crossed by the tracks of more slowly moving particles. It may be distinguished from them by means of the reversal of the Schlieren diaphragm which changes the color of the wave. Furthermore, the wave is reflected at the baffle. The curvature of the spherical shock-wave when it reaches the baffle is so small that the time interval between the wave touching the baffle and spreading around it is negligible. Hence, the trace of the wave beyond the baffle is a direct continuation of that on the near side; there is not any interference on the far side of the baffle due to the smoke particles.

The main body of products of combustion appears on both photographs as a black area. The impact of the reflected spherical shock-wave may temporarily check their advance.

Figure 5.—A charge of permissible explosive having a low rate of detonation was fired in one end of a cellophane tube with conditions of loading and size of tube similar to those employed with 40 per cent. straight dynamite. Two normal Schlieren photographs of the discharge of the explosive are shown in Figs. 5*A* and 5*B*. A narrow strip of paper was

FIG. 5.



wrapped around the tube 3.7 cm. from the open end in *A*, and in *B* the tube passed through a hole in a wooden screen 14.0 cm. from the end of the explosive charge. Two pieces of black paper were wrapped around the tube, one on each side of the screen.

The flame of the explosive has not the actinic qualities of 40 per cent. straight dynamite and the length of the flame is shorter. The plane-wave in the tube is propagated at an average velocity of 1250 meters per second along the whole length of the tube.

The plane shock-wave changes into a spherical shock-wave on emergence from the tube, and its velocity falls off continuously. In Fig. 5*B*, the spherical wave is reflected from a baffle placed near the edge of the field.

The cellophane tube was not immediately destroyed by the passage of the plane-wave in either experiment. In Fig. 5*A*, the portion of the tube beyond the paper strip was not appreciably moved; in Fig. 5*B*, the length of tube beyond the screen was recovered intact after the shot.

These two photographs show more clearly than Fig. 4, the formation of a vortex ring at the moment the plane-wave emerges from the tube mouth. The vortex ring has an average velocity in the field of view of 59 m.p.s. In Fig. 5*B*, the ring is not reflected from the baffle. A short time after striking the baffle, it breaks up and disappears.

The vortex ring is essentially the same as a smoke ring blown by a smoker. The fine suspended particles in the tobacco smoke, however, only serve to make the ring visible to the naked eye. The Schlieren apparatus is able to detect the ring, without such particles, by reason of the changed optical density which exists within the ring. This type of ring has also been observed and photographed at the open end of a small gallery employed in photographing gas-air ignitions caused by explosives.[15]

The spherical wave initiated close to the discharge of the explosive as indicated in the explanatory diagram at the side of Fig. 5 is propagated at an average velocity of 650 meters per second over a distance of 20.0 cm. from the end of the explosive charge. In Fig. 5*B* the spherical shock-wave is reflected from the screen with the exception of that portion which comes against the hole in the screen. This small portion passes through the hole and forms a spherical wave of much lower intensity on the other side of the screen.

Figure 6.—Further evidence as to the nature of the vortex ring is afforded by some photographs taken with the cellophane tube arranged at right angles to the stenopaic slit of the camera. The mouth of the tube was below the level of the slit, and no portion of the tube shows on the record.

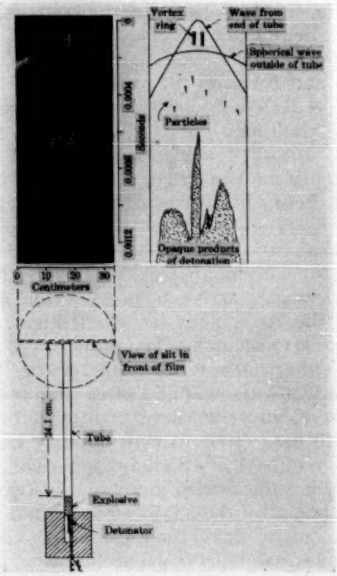
The explosive charge and method of loading were identical to those of the charge photographed in Fig. 5. A normal Schlieren photograph taken at the end of the cellophane tube is shown in Fig. 6. The distance between the end of the charge of explosive and the plane photographed through the camera

[15] Shepherd, W. C. F., unpublished report.

slit was 24.1 cm. The end of the tube was 6 mm. below the space photographed and does not appear in the record of the phenomena at the end of the tube.

The principal phenomena photographed in the order of their appearance are the wave from the end of the tube, vortex

FIG. 6.



ring, spherical wave outside of the tube, and the opaque products of detonation. The plane-wave passing through the cellophane tube becomes a spherical wave on emergence from the tube mouth. The spherical expansion in all directions from the end of the tube viewed through the slit in front of the camera traces the straight lines indicated in the diagram at the side of Fig. 7, which are a combination of the forward and lateral movements of the wave. One-half of the wave photo-

graphs with a normal Schlieren setting and the other half with a reversed Schlieren setting, because the lateral expansion is in both directions from the tube.

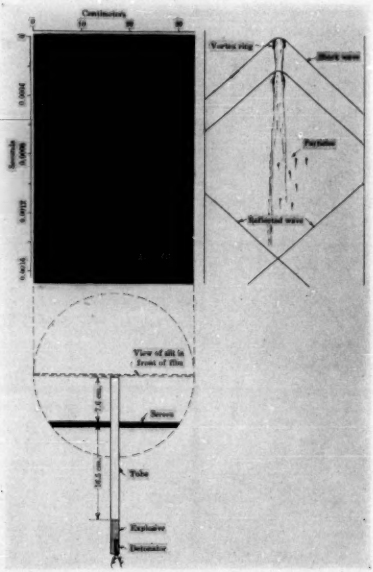
In general, photographs taken in this manner are symmetrical about a vertical dividing line, with the Schlieren effects of each half reversed in color.

The image caused by the passage of the ring across the slit will be found on the photograph by the help of the explanatory diagram. It consists of two bi-colored short bands, one on each half of the picture. A comparison of this photograph with Figs. 5*A* and *B* shows that the color of the outer line of the ring image is always reversed in color to that of the shock-wave, and that the inner line has the same color as the shock-wave. Simple optical considerations lead to the conclusion that the pressure in the vortex is less than atmospheric. Mathematical theory has shown this to be a necessary condition for the stability of such rings. The curved line represents a combination of the forward and lateral movements of the shock-wave. Measurement from Fig. 5*A* shows that the spherical shock-waves arrive 0.00018 second later than the plane-wave at the same point for which Fig. 6 gives the time difference as 0.00017 second. The plane-wave along the tube changes to a spherical wave on emergence from the tube. The main spherical wave will at a later time pass through this less intense wave, and the curve of intersection, as the two waves pass, will be a circle of continuously changing radius. The momentary impact of the main spherical shock-wave on the lesser wave is sufficient to increase abruptly the radius of this circle, as shown by the slight off-set when the two waves intersect. The main wave then passes on and both waves have their original velocity. Hence, during the passage of the main wave through the lesser wave, the latter is deformed and consists of two spherical portions joined by the circle of intersection, the one through which the main wave has passed having a somewhat larger radius of curvature than that of the other through which the wave is about to pass. The lesser wave will become spherical again as soon as the main wave has passed completely through.

Hot gases and pieces of cellophane shot out from the end of the tube leave traces appearing in the center of the photo-

graph after the passage of the spherical wave. Scattered particles passed in front of the slit in the camera at different times after the passage of the spherical wave. Opaque products of combustion which first appear directly above the end of the tube complete the phenomena photographed.

FIG. 7.



Figures 7 and 8.—Photographs of the plane-wave and vortex ring at the end of a cellophane tube are shown in Figs. 7 and 8. In these experiments, the spherical shock-wave and opaque products have been stopped by a large screen, through a hole in which the cellophane tube passed. The screen, as shown in the figure, is not in view of the camera. Figure 7 is a photograph taken across the end of the tube normal to its length and Fig. 8 is a photograph taken along the length of the tube.

The dimensions of the cellophane tube are shown in the figures, and, as distinct from the previous experiments, the charge of explosive and detonator were placed in the cellophane tube without the small cannon. The mirror was suitably protected from flying particles.

FIG. 8.

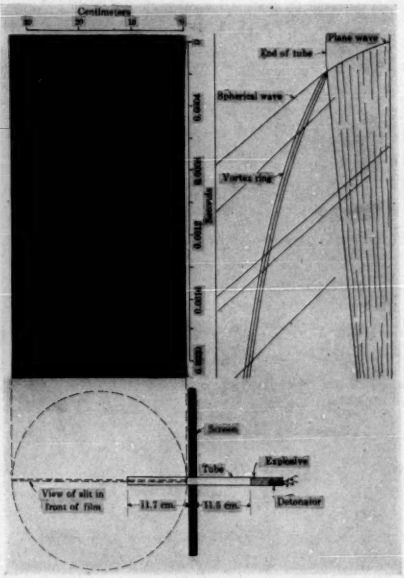


Figure 7 will be understood from the previous description of Fig. 6. The position of the tube was adjusted so that the mouth was at the same height as the stenopaic slit level in the field of view, and it will be noticed that the vortex ring follows immediately behind the wave. A second wave—due to that portion of the spherical wave which comes through the hole in the screen and travels up the cellophane tube as a plane-wave—and its attendant vortex ring, appear on the

second 0.0002 second later. These two waves are reflected from nearby objects and they reappear on the photographs about a thousandth of a second later.

Figure 8 is the record of a shot fired under identical conditions, photographed in the usual manner along the length of the tube. A short length of the tube extended into the field of view as shown in the diagram. The photograph shows the passage of the plane shock-wave along the tube and the formation of a vortex ring when it emerges from the mouth of the tube and becomes a spherical wave. The average speed of the spherical wave over the first 15 cm. of travel is 365 m.p.s. The vortex ring has a mean speed of 80 m.p.s. over the same distance, but this speed falls away rapidly to a low value.

SUMMARY.

The Schlieren method of photography, by means of which shock-waves and other phenomena are made visible, will be of great value in elucidating many explosive problems. Photographic evidence has been brought forward illustrating one of the many problems to which this apparatus may advantageously be applied. Speeds at any point of the shock-wave flame, particles of products are readily calculated, and their relative positions at all times are shown on the photograph. The records are permanent.

This preliminary study has been concerned only with the photography of the disturbance sent out by the detonation of small charges of explosive fired in the open and at one end of cellophane tubes. It forms part of a more complete study of gap tests.

The spherical shock-wave sent out by a small unconfined charge of explosive has initially a high speed of propagation. The speed of the wave falls off, rapidly at first, and then more slowly, until ultimately it will reach that of sound. The speed of projection of the products of detonation, under these conditions of firing, is low. Their advance may be temporarily arrested by a shock-wave travelling in the opposite direction. Solid particles break through the shock-wave a short distance from the original end of the explosive. Due to their inertia, their velocity of projection is maintained for some time. These particles appear to be projected some little time after

the shot is fired, but are able to overtake the shock-wave due to their own maintained velocity, and the rapid falling away of that of the wave.

The plane shock-wave (along the tube) sent out by a small charge of explosive, confined in one end of a uniform cellophane tube, maintains its high initial rate of propagation for considerable distances, with little loss of energy. A loss of energy. A loss of energy is made apparent by a reduction in speed. This plane shock-wave, in the case of 40 per cent. straight dynamite, has a constant rate of 2040 m.p.s. throughout the length of the tube; in the case of a permissible explosive, the speed was 1250 m.p.s. The plane shock-wave in the tube rapidly outstrips the spherical shock-wave of which it originally formed part, for the spherical wave is continuously losing speed and energy due to its expansion in all directions. This fact is interesting in connection with the effect of paper tubes on lengthening the maximum gas gap over which detonation is transmitted from one cartridge of explosive to another.

A vortex ring, invisible to the eye, is formed when the plane shock-wave emerges from the cellophane tube into the open. This ring is made visible to the Schlieren apparatus by reason of the decreased density within the ring. The velocity of the ring is small compared to that of the shock-wave which originates it. It has been shown that each successive wave coming out of the tube gives rise to a vortex ring.

The apparatus has been described in detail with the thought that this principle of photography may be readily applied to any problem in which small changes of optical density occur. The method of photography is not limited to phenomena occurring in gaseous systems.

DYNAMICAL ANALYSIS OF MACHINES.*

BY

R. EKSERGIAN, M.S., M.E.E., Ph.D.,

Consultant, Engineering Department,
E. I. DuPont de Nemours & Company,
Member of the Institute.

MACHINE WITH ELASTICALLY SUSPENDED FRAME (Fig. 76).

We will now consider a machine whose frame is elastically suspended. Many machines have this suspension at one point only, and are constrained by fixed bearings at the other. In practice we find examples in the Hotchkins' Drive in automobiles, in the usual spring suspension of the street car motor frame, etc. The problem is of considerable importance in the general theory of elastic frame suspension.

As an example completely illustrating the nature of this problem we will consider the reactions set up in a street car motor and its suspension. The main wheel axle has a gear wheel rigidly attached to it and which meshes with the pinion of the motor shaft. The motor and its pinion shaft are supported in bearings by the motor frame. The motor frame, in turn, is itself supported by bearings on the main axle and by a spring or elastic suspension at the truck frame. Thus the motor is partially spring supported which reduces the shocks resulting from poor track, etc.

If the nose suspension is rigid, the system is defined by the angular coördinate of the turning of the axle. Thus introducing an elastic suspension at the nose the armature frame itself can rotate, and we have a system with two degrees of freedom.

With a rigid suspension, the nose reaction R_s due to the traction loading is $F\frac{r}{l}$, where $F = \Phi\frac{\gamma}{r}$; hence $R_s = \Phi\frac{\gamma}{l}$,

* Extension of a portion of a dissertation for the degree of doctor of philosophy submitted to Clark University, 1928.

Continued from p. 658, vol. 210, November, 1930.

where F is the tangential force at the wheel and Φ is the motor torque with gear ratio γ. This is the load equilibrium condition for the spring. We will now investigate the dynamic loading and corresponding oscillation as a system with two degrees of freedom.

For simplification we will assume the wheel and motor axis to be in a horizontal plane.

Let

Φ = motor torque, $\gamma = \frac{r_g}{r_p}$ = motor gear ratio,

R_s = spring reaction of nose with moment arm l about the wheel axle,

F = tractive force at base of wheel,

F' = horizontal component at main journals of the reaction of the truck and car body.

M_m and I_m = mass and moment of inertia of motor.

M_c and I_c = mass and moment of inertia of the housing frame or casing, the latter being taken about the wheel axle,

h = c. of g. of casing from wheel axle and assumed located at motor pinion shaft, i.e., $h = r_g + r_p$.

q_r = relative angle turned by motor with respect to housing or casing.

q = absolute angle turned by motor.

θ = absolute angle turned by casing.

ϕ = absolute angle turned by driving wheel.

r = radius of wheel, r_g = radius of gear on axle, r_p = radius of pinion on motor shaft,

The system has two degrees of freedom, θ and ϕ. The condition relations are

$$r_g(\phi + \theta) = r_p q_r; \qquad \therefore \quad q_r = \gamma(\phi + \theta)$$

and

$$q = q_r + \theta = \theta + \gamma(\phi + \theta) = (1 + \gamma)\theta + \gamma\phi.$$

Therefore,

$$\frac{\partial q_r}{\partial \phi} = K_{qr\phi} = \gamma \qquad \text{and} \qquad \frac{\partial q_r}{\partial \theta} = K_{qr\theta} = \gamma,$$

$$\frac{\partial q}{\partial \phi} = K_{q\phi} = \gamma \qquad \text{and} \qquad \frac{\partial q}{\partial \theta} = K_{q\theta} = 1 + \gamma.$$

Further if x = horizontal displacement of axle and the system, then

$$x = r\phi; \qquad \therefore \quad K_{x\phi} = r,$$

and if y = the vertical displacement of motor and the c. of g. of casing,

$$y = h\theta; \qquad \therefore \quad K_{y\theta} = h.$$

For the virtual work of the applied forces, we have

$$\Phi\delta q_r - F'\delta x - [R_s l - (M_c + M_m)g \cdot h]\delta\theta$$
$$= \Phi\left(\frac{\partial q_r}{\partial \phi}\delta\phi + \frac{\partial q_r}{\partial \theta}\delta\theta\right) - F'\frac{\partial x}{\partial \phi}\delta\phi$$
$$- [R_s l - (M_c + M_m)gh]\delta\theta,$$

since the torque acts on the rotor and reacts on the motor housing through a total displacement equal to the relative displacement between them. Also the tangential force F being at an instantaneous center does no work and is omitted.

We have for the components of the motor torque corresponding to the coördinates ϕ and θ,

$$\Phi\frac{\partial q_r}{\partial \phi} = \gamma\Phi \qquad \text{and} \qquad \Phi\frac{\partial q_r}{\partial \theta} = \gamma\Phi,$$

so that

$$\Phi\frac{\partial q_r}{\partial \phi} = \Phi\frac{\partial q_r}{\partial \theta},$$

that is, the torque on the wheel system is equal and opposite to the torque on the housing since θ and ϕ are assumed in opposite directions, which, of course, could have been immediately anticipated from the principle of action and reaction. A detail proof from the consideration of internal forces is of interest. If R is the component tangential reaction at the contact of gear and pinion due to the motor torque, then

$R = \Phi/r_p$. Noting this reaction causes an equal reaction at the pinion shaft journals of the motor, we have

$Rr_g = \Phi\gamma$ = torque reaction on wheel system,

$-R(r_g + r_p) + \Phi = -Rr_g = -\Phi\gamma$ = torque reaction on housing.

The equations of motion are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) - \frac{\partial T}{\partial \phi} = \Phi\frac{\partial q_r}{\partial \phi} - F'\frac{\partial x}{\partial \phi},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} = \Phi\frac{\partial q_r}{\partial \theta} - R_s l + (M_c + M_m)gh,$$

where

$$T = \tfrac{1}{2}[M_m + M_c + M_w)\dot{x}^2 + M_m\dot{y}^2 + I_m\dot{q}^2 + I_c\dot{\theta}^2 + I_w\dot{\phi}^2]$$
$$= \tfrac{1}{2}[(M_m + M_c + M_w)r^2\dot{\phi}^2 + M_m h^2\dot{\theta}^2$$
$$+ I_m((1+\gamma)^2\dot{\theta}^2 + \gamma^2\dot{\phi}^2 + 2(1+\gamma)\gamma\dot{\theta}\dot{\phi}) + I_c\dot{\theta}^2 + I_w\dot{\phi}^2],$$

since

$$I_c = I_{c(cg)} + M_c h^2,$$

or, alternately,

$$(M_m + M_c + M_w)K_{x\phi}{}^2\ddot{\phi} + I_m(K_{q\phi}{}^2\ddot{\phi} + K_{q\phi}K_{q\theta}\ddot{\theta}) + I_w\ddot{\phi}$$
$$= \Phi K_{qr\phi} - F'K_{x\phi},$$

$$M_m K_{y\theta}{}^2\ddot{\theta} + I_m(K_{q\theta}{}^2\ddot{\theta} + K_{q\theta}K_{q\phi}\ddot{\phi}) + I_c\ddot{\theta}$$
$$= \Phi K_{qr\theta} - R_s l + (M_c + M_m)gh.$$

Proceeding by either method, we obtain

$$(M_t r^2 + I_m\gamma^2 + I_w)\ddot{\phi} + I_m(1+\gamma)\gamma\ddot{\theta} = \Phi\gamma - F'r, \quad (1)$$

$$(M_m h^2 + I_m(1+\gamma)^2 + I_c)\ddot{\theta} + I_m(1+\gamma)\ddot{\phi}$$
$$= \Phi\gamma - R_s l + (M_c + M_m)gh, \quad (2)$$

where $M_t = M_w + M_c + M_m$ = total mass of the system. Further, F', the reaction of the car body also can be expressed in terms of the acceleration ϕ, that is,

$$F' = M_b r\ddot{\phi} + R',$$

where M_b = the mass of the car body alone and R' = the translation resistance.

In the second expression the spring reaction is

$$R_s = cl\theta = cl(\theta_0 + \epsilon),$$

where

$$cl^2\theta_0 = \Phi_0\gamma + (M_c + M_m)gh$$

and

$$\Phi = \Phi_0 + \Delta\Phi.$$

Since

$$\ddot{\theta} = \frac{d^2}{dt^2}(\theta_0 + \epsilon) = \ddot{\epsilon},$$

we have for the equation of oscillation

$$\left[M_m h^2 + I_m(1+\gamma)^2 + I_c - \frac{\gamma^2(1+\gamma)^2 I_m^2}{A}\right]\ddot{\epsilon} + cl^2\epsilon = \Delta\Phi\gamma - \frac{\gamma(1+\gamma)I_m(\Phi\gamma - R'r)}{A},$$

where

$$A = M_t r^2 + I_m\gamma^2 + I_w + M_b r^2.$$

For the translation of the system, evidently,

$$F - F' = M_t r\ddot{\phi}. \qquad (4)$$

Subtracting (1) from (2), and noting the relation (4), we obtain

$$(M_m h^2 + I_c)\ddot{\theta} + I_m((1+\gamma)\ddot{\theta} + \gamma\ddot{\phi}) - I_w\ddot{\phi} = Fr - R_s l + (M_c + M_m)gh,$$

where

$$(1+\gamma)\ddot{\theta} + \gamma\ddot{\phi} = \ddot{q}.$$

This is obviously the equation of angular momentum about the axis of the wheel.

BEVEL GEAR DRIVE.

In the bevel gear drive power is transmitted through the propeller shaft and bevel gear to a transverse axle shaft or vice versa. When the axle housing and torque tube surrounding the propeller shaft are stationary it is evident that the torque reaction on the housing is equal and opposite to the impressed torque on the transverse axle shaft.

If now we consider a rotation of the housing and torque tube about the transverse axle shaft we have a system with two degrees of freedom.

Let

ψ = the absolute angle turned by the transverse axle shaft.

q = the rotation of the housing and torque tube about the transverse axle.

ϵ = the angular rotation of the propeller or motor shaft.

T_ϵ = the motor torque applied to the propeller shaft.

T_ψ = the torque on the transverse axle shaft.

T_q = the torque reaction on the housing and torque tube.

γ = the bevel gear ratio, etc.

$\sum mk^2$ = the moment of inertia of motor, flywheel, etc., about the propeller shaft.

$\sum ml^2$ = the moment of inertia of the propeller shaft, flywheel, etc., about the transverse axle shaft.

Consider now the system consisting of the propeller shaft, flywheel, etc., and the transverse axle shaft. The applied work forces are T_ϵ, T_ψ, and the reaction of the housing and torque tube on the propeller shaft bearings, manifested by the resisting moment T_q about the transverse axle shaft.

The rotation of the propeller shaft itself is affected by the rotation of the torque tube and housing about the transverse axis, that is,

$$\delta\epsilon = \gamma(\delta\psi + \delta q) \qquad \text{or} \qquad \dot{\epsilon} = \gamma(\dot{\psi} + \dot{q}).$$

The work of the engine torque is, therefore,

$$\gamma T_\epsilon(\delta\psi + \delta q),$$

and the work of the resisting torque on the transverse axle is $-T_\psi\delta\psi$, while the work of the housing reaction on the shafting is $-T_q\delta q$.

The generalized forces corresponding to the coördinates ψ and q are

$$\gamma T_\epsilon - T_\psi \qquad \text{and} \qquad \gamma T_\epsilon - T_q.$$

The kinetic energy of the shafting is

$$T = \tfrac{1}{2}\gamma^2 \Sigma mk^2(\dot{\psi} + \dot{q})^2 + \tfrac{1}{2}\Sigma ml^2 \cdot \dot{q}^2.$$

From the Lagrangian equations we obtain, noting the coördinates are of a cyclic nature,

$$\gamma T_s - T_\psi = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right),$$

$$\gamma T_s - T_q = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right),$$

or

$$\gamma T_s - T_\psi = \gamma^2 \Sigma mk^2(\ddot{\psi} + \ddot{q}),$$

$$\gamma T_s - T_q = \gamma^2 \Sigma mk^2(\ddot{\psi} + \ddot{q}) + \Sigma ml^2\ddot{q}.$$

Therefore,

$$T_\psi - T_q = \Sigma ml^2\ddot{q},$$

which is obviously the equation of angular momentum of the shafting about the transverse axle. Thus the reaction on the housing is less than the torque impressed on the transverse axle by the rate of change of angular momentum of the propeller shaft. If we neglect the angular momentum of the housing, then,

$$T_\psi = T_q = \gamma T_s - \gamma^2 \Sigma mk^2(\ddot{\psi} + \ddot{q}).$$

That is, the torque reaction on the housing is equal and opposite to the impressed torque on the transverse shafting.

RELAY MOTORS—OSCILLATING VEHICLE AXLE (Figs. 77–79).

As a further example of a system of two degrees of freedom, we will consider the dynamical equations for the Relay Oscillating Axle.

The load of the vehicle is not carried by the rear wheel center but is suspended from an oscillating arm which is free to swing about the axis of the rear wheel. Power is applied through a pinion at the lower end of the oscillating arm and meshes with a planetary gear attached to and concentric with the wheel center. By this construction the adhesion is augmented, torsional impacts are reduced, and the accelerating force is materially increased.

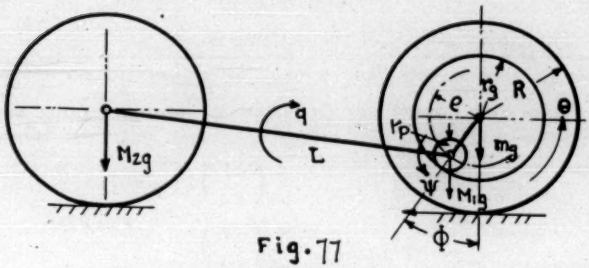


Fig. 77

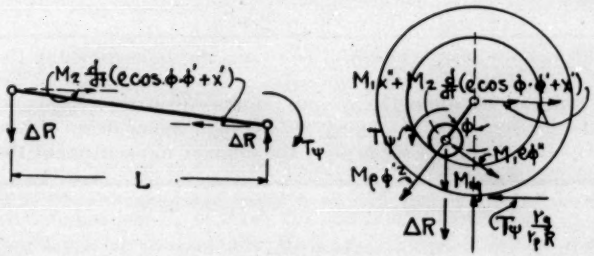


Fig. 78

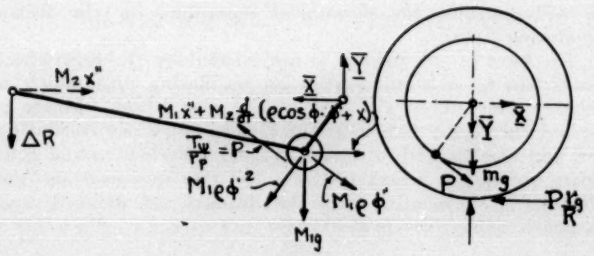


Fig. 79

The frame of the vehicle corresponds to the connecting link from the front axle to the pinion axis of the oscillating lever. For simplification we will neglect the obliquity of the connecting link and consider the mass of the vehicle as reduced (approximately) to two concentric masses at the oscillating pinion axis and the front wheel center, respectively.

Let

ρ = length of oscillating arm from axis of rear wheel to center of pinion,

r_p = radius of pinion gear,

r_g = radius of planetary gear attached to and concentric with wheel center,

R = radius of rear wheel,

L = length of connecting link of frame, i.e., the equivalent link connecting pinion axis at end of oscillating arm to front wheel center,

ψ = angle turned by pinion shaft,

ϕ = angle of swing of the oscillating arm,

θ = angle turned by rear wheel and concentric planetary gear,

x = corresponding translatory displacement of rear driving wheel,

M_1 = reduced mass for the rear part of vehicle at end of oscillating arm at pinion axis,

M_2 = reduced mass of front part of vehicle at front wheel axis, including wheel inertia,

m = mass of the rear driving wheel,

T_ψ = engine torque reduced to pinion axis,

$T_\star$ = engine torque, γ = gear ratio to pinion axis.

The equation of condition is

$$r_p\delta\psi - \rho\delta\phi = r_g\delta\theta$$

and the angular rotation of the connecting link or chassis frame is, neglecting its obliquity,

$$\delta q = \frac{\delta(\rho\cos\phi)}{L} = -\frac{\rho}{L}\sin\phi\cdot\delta\phi.$$

Further, if ϵ is the angular displacement of the engine shaft and γ the gear ratio to the pinion shaft, then the relative displacement of the engine shaft is

$$\delta\epsilon = \gamma(\delta\psi + \delta q),$$

which shows the rotation of the propeller shaft about a longitudinal axis is affected by the rotation of the frame about a transverse axis.

The work of the engine torque T_e is

$$T_e\delta\epsilon = \gamma T_e(\delta\psi + \delta q).$$

If $\sum mk^2$ is the rotational inertia coefficient of the engine, flywheel, etc., the kinetic energy is

$$T_l = \tfrac{1}{2}\sum m_e k_e^2\gamma^2(\dot{\psi} + \dot{q})^2 \,[1]$$

The reduced torques at the pinion shaft acting on the coördinates ψ and q are

$$T_\psi = \gamma T_e - \frac{d}{dt}\frac{\partial T_l}{\partial\dot{\psi}} = \gamma T_e - \gamma^2\sum m_e k_e(\ddot{\psi} + \ddot{q}),$$

$$T_q = \gamma T_e - \frac{d}{dt}\frac{\partial T_l}{\partial\dot{q}} = \gamma T_e - \gamma^2\sum m_e k_e^2(\ddot{\psi} + \ddot{q}).$$

Therefore, $T_\psi = T_q$, that is, the torque on the pinion shaft is equal to the torque reaction on the frame housing at the pinion shaft, which, of course, is obvious from the principle of action and reaction.

By means of the condition relations, the work of the reduced engine torque T_ψ at the pinion shaft can be expressed in terms of the coördinates θ and ϕ, that is,

$$T_\psi(\delta\psi + \delta q) = T_\psi\frac{r_g}{r_p}\delta\theta + T_\psi\left(\frac{\rho}{r_p} - \frac{\rho}{L}\sin\phi\right)\delta\phi.$$

If P is the gear reaction at the pinion shaft, we have $Pr_p = T_\psi$, so that, also,

$$T_\psi(\delta\psi + \delta q) = Pr_g\delta\theta + P\frac{\rho}{L}(L - r_p\sin\phi)\delta\phi,$$

[1] The angular momentum of the propeller shaft, etc., about the transverse pinion shaft is neglected.

but

$$\delta\theta = \frac{\delta x}{R},$$

so that

$$\delta W = T_\psi \frac{r_g}{r_p}\frac{\delta x}{R} + T_\psi\left(\frac{\rho}{r_p} - \frac{\rho}{L}\sin\phi\right)\delta\phi$$

$$= P\frac{r_g}{R}\delta x + P\frac{\rho}{L}(L - r_p\sin\phi)\delta\phi.$$

The generalized reactions, corresponding to the coördinates x and ϕ are, therefore,

$$T_\psi\frac{r_g}{Rr_p} \quad \text{and} \quad T_\psi\left(\frac{\rho}{r_p} - \frac{\rho}{L}\sin\phi\right) - M_1 g\sin\phi\cdot\rho,$$

or in terms of the pinion reaction,

$$P\frac{r_g}{R} \quad \text{and} \quad P\frac{\rho}{L}(L - r_p\sin\phi) - M_1 g\sin\phi\cdot\rho.$$

The rear end of the vehicle M_1 is subjected to a lift $\delta(\rho\cos\phi)$ and a translation $\delta(\rho\sin\phi + x)$, while the front part, M_2, has merely the translation component $\delta(\rho\sin\phi + x)$. If m is the mass of the wheel, the velocity components are

Lift velocity	$= -\rho\sin\phi\cdot\dot\phi,$	for M_1,
Translation velocity	$= \rho\cos\phi\cdot\dot\phi + \dot x,$	for M_1,
Translation velocity	$= \rho\cos\phi\cdot\dot\phi + \dot x,$	for M_2,
Translation velocity for m	$= \dot x.$	

The total kinetic energy of the system is, therefore,

$$T = \tfrac{1}{2}m\dot x^2 + \tfrac{1}{2}M_1[\rho^2\sin^2\phi\cdot\dot\phi^2 + (\rho^2\cos^2\phi\cdot\dot\phi^2 + 2\rho\cos\phi\cdot\dot\phi\dot x + \dot x^2)] + \tfrac{1}{2}M_2(\rho^2\cos^2\phi\cdot\dot\phi^2 + 2\rho\cos\phi\cdot\dot\phi\dot x + \dot x^2)$$

or

$$T = \tfrac{1}{2}(m + M_1 + M_2)\dot x^2 + \tfrac{1}{2}(M_1 + M_2\cos^2\phi)\rho^2\dot\phi^2 + (M_1 + M_2)\rho\cos\phi\cdot\dot\phi\dot x.$$

From the Lagrangian equations, we have

$$\frac{\partial T}{\partial\dot x} = (m + M_1 + M_2)\dot x + (M_1 + M_2)\rho\cos\phi\cdot\dot\phi,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = (m + M_1 + M_2)\ddot{x} + (M_1 + M_2)\rho \cos\phi \cdot \ddot{\phi} - (M_1 + M_2)\rho \sin\phi \cdot \dot{\phi}^2$$

and

$$\frac{\partial T}{\partial x} = 0,$$

$$\frac{\partial T}{\partial \dot{\phi}} = (M_1 + M_2 \cos^2\phi)\rho^2\dot{\phi} + (M_1 + M_2)\rho \cos\phi \cdot \dot{\phi},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) = (M_1 + M_2 \cos^2\phi)\rho^2\ddot{\phi} - 2M_2 \sin\phi \cos\phi\rho^2\dot{\phi}^2 \cdot - (M_1 + M_2)\rho \sin\phi \cdot \dot{\phi}\dot{x} + (M_1 + M_2)\rho \cos\phi \cdot \ddot{x}$$

$$\frac{\partial T}{d\phi} = -M_2 \sin\phi \cos\phi\rho^2\dot{\phi}^2 - (M_1 + M_2)\rho \sin\phi \cdot \dot{\phi}\dot{x}.$$

Therefore, neglecting the friction components, the equations of motion are

$$(m + M_1 + M_2)\ddot{x} + (M_1 + M_2)\rho \cos\phi \cdot \ddot{\phi} - (M_1 + M_2)\rho \sin\phi \cdot \dot{\phi}^2 = T_\psi \frac{r_g}{Rr_p} = P\frac{r_g}{R} \quad (1)$$

and

$$(M_1 + M_2 \cos^2\phi)\rho^2\ddot{\phi} + (M_1 + M_2)\rho \cos\phi - M_2 \sin\phi \cos\phi\rho^2 \cdot \dot{\phi}^2 = T_\psi\left(\frac{\rho}{r_p} - \frac{\rho}{L}\sin\phi\right) - M_1 g \sin\phi \cdot \rho$$

$$= P\frac{\rho}{L}(L - r_p \sin\phi) - M_1 g \sin\phi \cdot \rho. \quad (2)$$

We may verify these equations from the principles of linear and angular momentum as follows (Fig. 78):

The external force on the entire system is the traction at the base of the wheels, i.e.,

$$T_\psi \frac{r_g}{Rr_p} = P\frac{r_g}{R}.$$

The acceleration in the direction of traction is, for $m\ddot{x}$ and for M_1 and M_2,

$$\frac{d}{dt}(\rho \cos\phi \cdot \dot{\phi} + \dot{x}) = \rho \cos\phi \cdot \ddot{\phi} + \ddot{x} - \rho \sin\phi \cdot \dot{\phi}^2,$$

so that eq. (1) is also the rate of change of the linear momentum for the system.

For the second equation we will take moments about the axis of the driving wheels which it is important to note is subjected to an acceleration $\ddot{x}$. The rate of change of momentum about this axis is the moment of the kinetic reactions and applied forces. The reactions on the wheel system and oscillating arm and pinion consist of the kinetic reactions of M_1 divided into a translation component $M_1\ddot{x}$, a centrifugal component $M_1\rho\dot{\phi}^2$ and a tangential component $M_1\rho\ddot{\phi}$ and its weight, M_1g. The reaction of M_2 is the translation component, $M_2(d/dt)(\rho\cos\phi\cdot\dot{\phi}+\dot{x})$. The applied forces consist of the reduced engine torque T_ψ acting on the pinion and the traction and vertical component at the wheel base. Due to the torque reaction on the frame, we have the supporting force on the frame at the pinion axis increased by

$$\Delta R = \frac{T_\psi}{L},$$

which causes a corresponding downward reaction on the oscillating arm at the pinion axis.

If now we consider only the wheel and planetary gear (Fig. 79), the turning moment is $P_\psi(r_g/r_p)$, which causes a balancing tangential reaction at the wheel tread,

$$T_\psi\frac{r_g}{Rr_p} = P\frac{r_g}{R}.$$

Taking moments about the rear axle for the entire wheel system and oscillating arm and pinion, including the moments of the kinetic reactions, we have

$$T_\psi\frac{r_g}{Rr_p}\cdot R - T_\psi - M_2(\rho\cos\phi\cdot\ddot{\phi} + \ddot{x} - \rho\sin\phi\cdot\dot{\phi}^2)\rho\cos\phi$$
$$- M_1\ddot{x}\rho\cos\phi - M_1\rho^2\ddot{\phi} - M_1g\rho\sin\phi - \Delta R\cdot\rho\sin\phi = 0;$$

$$\therefore\quad (M_1 + M_2\cos^2\phi)\rho^2\ddot{\phi} + (M_1 + M_2)\ddot{x}\rho\cos\phi$$
$$- M_2\rho^2\sin\phi\cos\phi\cdot\dot{\phi}^2 = T_\psi\left(\frac{\rho}{r_p} - \frac{\rho}{L}\sin\phi\right) - M_1g\rho\sin\phi,$$

which verifies the previous derivation.

An alternative method of the resolution of the reactions would be by considering separately the planetary gear and wheel, and the frame with the oscillating link and its pinion together. The mutual reaction between the systems is then the tangential gear reaction P and the component reactions at the wheel axis and fulcrum of the oscillating arm. In taking moments about the wheel axis we are not concerned with the latter.

In considering the system of the frame and oscillating link together it is important to determine the change in loading ΔR at the front wheel axis. This is readily obtained by considering the frame above with the pinion gear. Noting the moment of the oscillating link about the pinion axis is nil, we have for moments about the axis of the pinion

$$P r_p = T_\psi = \Delta R \cdot L; \qquad \therefore \quad \Delta R = \frac{T_\psi}{L}.$$

Then, taking moments about the wheel axis for the system consisting of frame and oscillating link, we have, for the moments of the applied forces (noting the weight $M_2 g$ is balanced by its corresponding statical wheel support at the front wheel),

$$P(\rho + r_p) - \Delta R(L + \rho \sin \phi) - M_1 g \rho \sin \phi = T_\psi \left(\frac{\rho}{r_p} - \frac{\rho}{L} \sin \phi \right) - M_1 g \rho \sin \phi,$$

agreeing with the moment of the applied forces from the last equation. The moment of the kinetic reactions are obviously the same as before.

Finally, we might have considered the entire system, noting that due to the torque reaction at the shaft, $\Delta R = T_\psi / L$. In this case we must also consider the moment of the tangential force at the rear wheel base. We have, then, for the moment of the applied forces about the rear wheel axis,

$$T_\psi \frac{r_g}{r_p} - \Delta R(L + \rho \sin \phi) - M_1 g \rho \sin \phi = T_\psi \left(\frac{\rho}{r_p} - \frac{\rho}{L} \sin \phi \right) - M_1 g \rho \sin \phi.$$

A more complete analysis of this problem should include the actual inertia of the suspended vehicle rather than the approximation of concentrated masses. Then ΔR would be decreased over the above due to the compensating effect of the kinetic reactions resulting from the angular acceleration of the frame. In the above analysis this component is entirely neglected due to the fact that the angular displacement q is at best small.

Analysis by Method of Velocity Ratios (Fig. 77).—The equations of condition for the oscillating axle are

$$r_p\delta\psi - \rho\delta\phi = r_g\delta\theta = r_g\frac{\delta x}{R}$$

and

$$\delta q = \frac{\delta(\rho\cos\phi)}{L} = -\frac{\rho}{L}\sin\phi\delta\phi,$$

so that

$$\delta\psi = \frac{\rho}{r_p}\delta\phi + \frac{r_g}{r_p}\frac{\delta x}{R},$$

$$K_{\psi\phi} = \frac{\rho}{r_p}, \qquad K_{\psi x} = \frac{r_g}{Rr_p}, \qquad \text{and} \qquad K_{q\phi} = -\frac{\rho}{L}\sin\phi.$$

Velocity Ratios for M_1:

$$\delta x_1 = \rho\delta\phi\cos\phi + \delta x,$$
$$\delta y_1 = -\rho\delta\phi\sin\phi,$$

$$K_{x_1\phi} = \rho\cos\phi, \qquad \frac{\partial K_{x_1\phi}}{\partial\phi} = -\rho\sin\phi,$$

$$K_{x_1x} = 1.$$

$$K_{y_1\phi} = -\rho\sin\phi, \qquad \frac{\partial K_{y_1\phi}}{\partial\phi} = -\rho\cos\phi,$$

Velocity Ratios for M_2:

$$\delta x_2 = \rho\delta\phi\cos\phi + \delta x,$$

$$K_{x_2\phi} = \rho\cos\phi, \qquad \frac{\partial K_{x_2\phi}}{\partial\phi} = -\rho\sin\phi, \qquad K_{x_2x} = 1.$$

For the work of the applied forces we have

$$T_\psi(\delta\psi + \delta q) + M_1g\delta y_1 = T_\psi(K_{\psi\phi}\delta\phi + K_{\psi x}\delta x) + T_\psi K_{q\phi}\delta\phi + M_1gK_{y_1\phi}\delta\phi,$$

so that the applied forces for ϕ and x are

$$T_\psi(K_{\psi\phi} + K_{g\phi}) + M_1 g K_{y_1\phi} \quad \text{and} \quad T_\psi K_{\psi x}.$$

The equations of motion are

$$M_1\left[(K_{x_1\phi}^2 + K_{y_1\phi}^2)\phi + K_{x_1\phi}K_{x_1x}\ddot{x} + \left(K_{x_1\phi}\frac{\partial K_{x_1\phi}}{\partial\phi} + K_{y_1\phi}\frac{\partial K_{y_1\phi}}{\partial\phi} \right)\dot{\phi}^2 \right]$$

$$+ M_2\left[K_{x_2\phi}^2\ddot{\phi} + K_{x_2\phi}K_{x_2x}\ddot{x} + K_{x_2\phi}\frac{\partial K_{x_2\phi}}{\partial\phi}\cdot\dot{\phi}^2 \right] = T_\psi(K_{\psi\phi} + K_{g\phi}) + M_1 g K_{y_1\phi}$$

and

$$M_1\left[K_{x_1x}^2\ddot{x} + K_{x_1x}K_{x_1\phi}\ddot{\phi} + K_{x_1x}\frac{\partial K_{x_1\phi}}{\partial\phi}\cdot\dot{\phi}^2 \right]$$

$$+ M_2\left[K_{x_2x}^2\ddot{x} + K_{x_2x}K_{x_2\phi}\ddot{\phi} + K_{x_2x}\frac{\partial K_{x_2\phi}}{\partial\phi}\cdot\dot{\phi}^2 \right] + m\ddot{x} = T_\psi K_{\psi x}.$$

Substituting for the velocity ratios, we obtain

$$(M_1 + M_2\cos^2\phi)\rho^2\ddot{\phi} + (M_1 + M_2)\rho\cos\phi\cdot\ddot{x} - M_2\rho^2\sin\phi\cos\phi\cdot\dot{\phi}^2 = T_\psi\left(\frac{\rho}{r_p} - \frac{\rho}{L}\sin\phi\right) - M_1 g\rho\sin\phi,$$

$$(M_1 + M_2 + m)\ddot{x} + (M_1 + M_2)\rho\cos\phi\cdot\ddot{\phi} - (M_1 + M_2)\rho\sin\phi\cdot\dot{\phi}^2 = T_\psi\frac{r_g}{Rr_p}.$$

For the translation of the system we have

$$F = m\ddot{x} + M_1\left[K_{x_1x}\ddot{x} + K_{x_1\phi}\ddot{\phi} + \frac{\partial K_{x_1\phi}}{\partial\phi}\cdot\dot{\phi}^2 \right] + M_2\left[K_{x_2x}\ddot{x} + K_{x_2\phi}\ddot{\phi} + \frac{\partial K_{x_2\phi}}{\partial\phi}\cdot\dot{\phi}^2 \right];$$

$$\therefore \quad F = (m + M_1 + M_2)\ddot{x} + (M_1 + M_2)\rho\cos\phi\cdot\ddot{\phi} - (M_1 + M_2)\rho\sin\phi\cdot\dot{\phi}^2,$$

where F = the tangential friction force at the base of the driving wheel $= T_\psi K_{\psi x} = T_\psi(r_g/Rr_p)$, neglecting the small rotational inertia of the wheel itself.

DYNAMICS OF A MECHANISM ON A ROTATING FRAME.

In dealing with a mechanism attached to a rotating frame we are concerned with the relative coördinates, x_r, y_r, with respect to the frame which can be expressed as functions of generalized relative coördinates α, β, etc. The absolute coordinates, x, y, are related with the relative coördinates and the angle ψ turned by the frame, that is, in the most general form, we have the linear relations

$$x = (a + x_r)\cos\psi - (b + y_r)\sin\psi,$$
$$y = (a + x_r)\sin\psi + (b + y_r)\cos\psi,$$

and

$$x_r = x\cos\psi + y\sin\psi - a,$$
$$y_r = y\cos\psi - x\sin\psi - b.$$

Therefore, the constraints of the mechanism which are condition relations between the relative coördinates, can also be expressed in terms of the absolute coördinates and the angle turned by the frame, that is,

$$F(x_r, y_r) = F(\alpha, \beta) = F(x, y, \psi) = 0, \text{ etc.}$$

From D'Alembert's equation and the above constraints we have

$$\Sigma\left(m\ddot{x} - X + \lambda\frac{\partial F}{\partial x}\right)\delta x + \Sigma\left(m\ddot{y} - Y + \lambda\frac{\partial F}{\partial y}\right)\delta y + \left(I_f\ddot{\psi} - \Phi + \lambda\frac{\partial F}{\partial\psi}\right)\delta\psi = 0,$$

where I_f is the polar moment of inertia of the frame and Φ the applied torque of the shaft on the frame.

If we choose λ so that the coefficient of the last term is nil, due to the independence of the therefore arbitrary variations of x and y these coefficients must likewise be zero, so that

$$\left.\begin{aligned}\Sigma m\ddot{x} - X + \lambda\frac{\partial F}{\partial x} &= 0\\ \Sigma m\ddot{y} - Y + \lambda\frac{\partial F}{\partial y} &= 0\end{aligned}\right\} \quad \text{and} \quad I_f\ddot{\psi} - \Phi + \lambda\frac{\partial\psi}{\partial F} = 0.$$

Evidently $\lambda \frac{\partial F}{\partial \psi}$ is the reaction of the mechanism on the rotating frame. This can also be shown from the following consideration. The work of the reactions of the constraint on the mechanism is, also,

$$\delta W_c = \lambda \frac{\partial F}{\partial x} \delta x + \lambda \frac{\partial F}{\partial y} \delta y.$$

Noting x and y are functions of x_r, y_r, and ψ, then

$$\begin{aligned} \delta W_c = {} & \lambda \left(\frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial x_r} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial x_r} \right) \delta x_r \\ & + \lambda \left(\frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial y_r} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial y_r} \right) \delta y_r \\ & + \lambda \left(\frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial \psi} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial \psi} \right) \delta \psi; \end{aligned}$$

$$\therefore \quad \delta W_c = \lambda \frac{\partial F}{\partial x_r} \partial x_r + \lambda \frac{\partial F}{\partial y_r} dy_r - \lambda \frac{\partial F}{\partial \psi} \delta \psi.$$

Now since $F(x_r y_r) = 0$,

$$\therefore \quad \delta W_c = - \lambda \frac{\partial F}{\partial \psi} \delta \psi.$$

The equation of virtual work for the mechanism is

$$\Sigma(m\ddot{x} - X)\delta x + \Sigma(m\ddot{y} - Y)\delta y = \lambda \frac{\partial F}{\partial \psi} \delta \psi.$$

In an actual mechanism we may consider the distributed mass points as resolved into concentrated mass points m, and the inertia moments of the members I. Let ϕ = the angle turned by any member, and ϕ_r the angle turned relative to the frame. Then $\phi = \phi_r + \psi$, and the equation of virtual work is

$$\Sigma(m\ddot{x} - X)\delta x + \Sigma(m\ddot{y} - Y)\delta y + \Sigma(I\phi - L)\delta\phi = \lambda \frac{\partial F}{\partial \psi} \delta \psi ,$$

or in terms of the relative coördinates and the angle ψ,

$$\Sigma(m\ddot{x} - X)\left(\frac{\partial x}{\partial x_r}\delta x_r + \frac{\partial x}{\partial y_r}\delta y_r\right)$$
$$+ \Sigma(m\ddot{y} - Y)\left(\frac{\partial y}{\partial x_r}\delta x_r + \frac{\partial y}{\partial y_r}\delta y_r\right) + \Sigma(I\ddot{\phi} - L)\delta\phi_r$$
$$= \left[\left(X\frac{\partial x}{\partial \psi} + Y\frac{\partial y}{\partial \psi} + L + \lambda\frac{\partial F}{\partial \psi}\right)\right.$$
$$\left. - \left(\Sigma m\ddot{x}\frac{\partial x}{\partial \psi} + \Sigma m\ddot{y}\frac{\partial y}{\partial \psi} + \Sigma I\ddot{\phi}\right)\right]\delta\psi.$$

From the relations of the coördinates, we note $\frac{\partial x}{\partial \psi} = -y$ and $\frac{\partial y}{\partial \psi} = x$, so that from the equation of angular momentum the expression in brackets on the right side of the above equation must be nil. Therefore, the dynamical equation of a rotating frame system is

$$\Sigma(m\ddot{x} - X)\left(\frac{\partial x}{\partial x_r}\delta x_r + \frac{\partial x}{\partial y_r}\delta y_r\right)$$
$$+ \Sigma(m\ddot{y} - Y)\left(\frac{\partial y}{\partial x_r}\delta x_r + \frac{\partial y}{\partial y_r}\delta y_r\right) + \Sigma(I\ddot{\phi} - L)\delta\phi_r = 0.$$

Some simplification is obtained by assuming the relative and fixed axes to coincide at the instant under consideration. Then, in the linear relations, $\cos\psi = 1$ and $\sin\psi = 0$, so that

$$\frac{\partial x}{\partial y_r} = \frac{\partial y}{\partial x_r} = 0 \quad \text{and} \quad \frac{\partial x}{\partial x_r} = \frac{\partial y}{\partial y_r} = 1.$$

Hence,

$$\Sigma(m\ddot{x} - X)\delta x_r + \Sigma(m\ddot{y} - Y)\delta y_r + \Sigma(I\ddot{\phi} - L)\delta\phi_r = 0.$$

If we express the relative coördinates in terms of the independent generalized relative coördinates α and β, then

$$\Sigma(m\ddot{x} - X)\frac{\partial x_r}{\partial \alpha} + \Sigma(m\ddot{y} - Y)\frac{\partial y_r}{\partial \alpha} + \Sigma(I\ddot{\phi} - L)\frac{\partial \phi_r}{\partial \alpha} = 0,$$

$$\Sigma(m\ddot{x} - X)\frac{\partial x_r}{\partial \beta} + \Sigma(m\ddot{y} - Y)\frac{\partial y_r}{\partial \beta} + \Sigma(I\ddot{\phi} - L)\frac{\partial \phi_r}{\partial \beta} = 0.$$

The accelerations can be expressed in terms of the relative coördinates and the angle of rotation of the frame, that is,

$$\ddot{x} = \frac{\partial x}{\partial \alpha}\ddot{\alpha} + \frac{\partial x}{\partial \beta}\ddot{\beta} + \frac{\partial x}{\partial \psi}\ddot{\psi} + \frac{\partial^2 x}{\partial \alpha^2}\dot{\alpha}^2 + \frac{\partial^2 x}{\partial \beta^2}\dot{\beta}^2 + \frac{\partial^2 x}{\partial \psi^2}\dot{\psi}^2 + 2\frac{\partial^2 x}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + 2\frac{\partial^2 x}{\partial \alpha \partial \psi}\dot{\alpha}\dot{\psi} + 2\frac{\partial^2 x}{\partial \beta \partial \psi}\dot{\beta}\dot{\psi},$$

$$\ddot{y} = \frac{\partial y}{\partial \alpha}\ddot{\alpha} + \frac{\partial y}{\partial \beta}\ddot{\beta} + \frac{\partial y}{\partial \psi}\ddot{\psi} + \frac{\partial^2 y}{\partial \alpha^2}\dot{\alpha}^2 + \frac{\partial^2 y}{\partial \beta^2}\dot{\beta}^2 + \frac{\partial^2 y}{\partial \psi^2}\dot{\psi}^2 + 2\frac{\partial^2 y}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + 2\frac{\partial^2 y}{\partial \alpha \partial \psi}\dot{\alpha}\dot{\psi} + 2\frac{\partial^2 y}{\partial \psi \partial \beta}\dot{\beta}\dot{\psi},$$

$$\ddot{\phi} = \frac{\partial \phi}{\partial \alpha}\ddot{\alpha} + \frac{\partial \phi}{\partial \beta}\ddot{\beta} + \frac{\partial^2 \phi}{\partial \alpha^2}\dot{\alpha}^2 + \frac{\partial^2 \phi}{\partial \beta^2}\dot{\beta}^2 + 2\frac{\partial^2 \phi}{\partial \alpha \partial \beta}\dot{\alpha}\dot{\beta} + \ddot{\psi}.$$

When the rotating and fixed axes are coincident,

$$\frac{\partial x}{\partial \alpha} = \frac{\partial x_r}{\partial \alpha} = K_{x\alpha}, \qquad \frac{\partial x}{\partial \beta} = \frac{\partial x_r}{\partial \beta} = K_{x\beta}, \qquad \frac{\partial x}{\partial \psi} = -y = -y_r,$$

$$\frac{\partial y}{\partial \alpha} = \frac{\partial y_r}{\partial \alpha} = K_{y\alpha}, \qquad \frac{\partial y}{\partial \beta} = \frac{\partial y_r}{\partial \beta} = K_{y\beta}, \qquad \frac{\partial y}{\partial \psi} = x = x_r,$$

$$\frac{\partial^2 x}{\partial \psi^2} = -\frac{\partial y}{\partial \psi} = -x_r, \qquad \frac{\partial^2 y}{\partial \psi^2} = \frac{\partial x}{\partial \psi} = -y_r,$$

$$\frac{\partial^2 x}{\partial \alpha \partial \psi} = -\frac{\partial y}{\partial \alpha} = -\frac{\partial y_r}{\partial \alpha} = -K_{y\alpha}, \qquad \frac{\partial^2 x}{\partial \beta \partial \psi} = -K_{y\beta},$$

$$\frac{\partial^2 y}{\partial \alpha \partial \psi} = \frac{\partial x}{\partial \alpha} = \frac{\partial x_r}{\partial \alpha} = K_{x\alpha}, \qquad \frac{\partial^2 y}{\partial \beta \partial \psi} = K_{x\beta},$$

and

$$\frac{\partial \phi}{\partial \alpha} = \frac{\partial \phi_r}{\partial \alpha} = K_{\phi\alpha}, \qquad \frac{\partial \phi}{\partial \beta} = K_{\phi\beta},$$

where $K_{x\alpha}$, $K_{x\beta}$, etc., are the partial relative velocity ratios of the mechanism with respect to the rotating frame.

The equations of motion are, therefore,

$$\Sigma m\Big[(K_{x\alpha}{}^2 + K_{y\alpha}{}^2)\ddot{\alpha} + (K_{x\alpha}K_{x\beta} + K_{y\alpha}K_{y\beta})\ddot{\beta} - (y_r K_{x\alpha} - x_r K_{y\alpha})\ddot{\psi} + \left(K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial \alpha} + K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial \alpha} \right)\dot{\alpha}^2 + \left(K_{x\alpha}\frac{\partial K_{x\beta}}{\partial \beta} + K_{y\alpha}\frac{\partial K_{y\beta}}{\partial \beta} \right)\dot{\beta}^2 - (x_r K_{x\alpha} + y_r K_{y\alpha})\dot{\psi}^2$$

$$+ 2\left(K_{x\alpha}\frac{\partial K_{x\beta}}{\partial\alpha} + K_{y\alpha}\frac{\partial K_{y\beta}}{\partial\alpha}\right)\dot{\alpha}\dot{\beta} - 2(K_{x\alpha}K_{y\beta} - K_{y\alpha}K_{x\beta})\dot{\beta}\dot{\psi}\Big]$$

$$+\Sigma I\left(K_{\phi\alpha}{}^2\ddot{\alpha} + K_{\phi\alpha}K_{\phi\beta}\ddot{\beta} + K_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\partial\alpha}\dot{\alpha}^2 + K_{\phi\alpha}\frac{\partial K_{\phi\beta}}{\partial\beta}\dot{\beta}^2\right.$$

$$\left. + 2K_{\phi\alpha}\frac{\partial K_{\phi\beta}}{\partial\alpha}\dot{\alpha}\dot{\beta}\right) = \Sigma(XK_{x\alpha} + YK_{y\alpha} + LK_{\phi\alpha}) - \frac{\partial V}{\partial\alpha},$$

with a similar equation for the coördinate β.

For one degree of freedom and uniform rotation of the frame, the equation reduces to the simple form,

$$[\Sigma m(K_{x\alpha}{}^2 + K_{y\alpha}{}^2) + \Sigma IK_{\phi\alpha}{}^2]\ddot{\alpha}$$

$$+\left[\Sigma m\left(K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial\alpha} + K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial\alpha}\right) + \Sigma IK_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\partial\alpha}\right]\dot{\alpha}^2$$

$$= \Sigma(XK_{x\alpha} + YK_{y\alpha} + LK_{\phi\alpha}) + \Sigma m(x_rK_{x\alpha} + y_rK_{y\alpha})\dot{\psi}^2,$$

where the last term is the reduced centrifugal forces.

If, on the other hand, we consider the frame constrained by a torsional spring and therefore subjected to small oscillations, the centrifugal terms of the frame may be omitted and we have

$$[\Sigma m(K_{x\alpha}{}^2 + K_{y\alpha}{}^2) + \Sigma IK_{\phi\alpha}{}^2]\ddot{\alpha}$$

$$+\left[\Sigma m\left(K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial\alpha} + K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial\alpha}\right) + \Sigma IK_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\partial\alpha}\right]\dot{\alpha}^2$$

$$= \Sigma(XK_{x\alpha} + YK_{y\alpha} + LK_{\phi\alpha}) + \Sigma m(y_rK_{x\alpha} - x_rK_{y\alpha})\ddot{\psi}.$$

We noted from the equation of angular momentum the reaction on the frame is

$$\lambda\frac{\partial F}{\partial\psi} = \Sigma m\ddot{x}\frac{\partial x}{\partial\psi} + \Sigma m\ddot{y}\frac{\partial y}{\partial\psi} + \Sigma I\ddot{\phi} - \Sigma\left(X\frac{\partial x}{\partial\psi} + Y\frac{\partial y}{\partial\psi} + L\right)$$

or

$$\lambda\frac{\partial F}{\partial\psi} = \Sigma m\ddot{y}\cdot x - \Sigma m\ddot{x}\cdot y + \Sigma I\ddot{\phi} - (Yx - Xy + L).$$

The forces and couples X, Y, and L are applied work forces on the mechanism. If they are of an internal character, as the mutual steam thrust on the piston and cylinder head

of a cylinder or the mutual torque reaction between parts of the mechanism as in a driving motor, then in the equation of momentum

$$\Sigma\left(X\frac{\partial x}{\partial\psi}+Y\frac{\partial y}{\partial\psi}+L\right)=\Sigma(Yx-Xy+L)=0.$$

For the motion of the frame,

$$\Phi-\lambda\frac{\partial F}{\partial\psi}=I_f\ddot{\psi}.$$

If we assume a torsional spring attached to the frame, then, $\Phi=-c\psi$, so that

$$\lambda\frac{\partial F}{\partial\psi}=-c\psi-I_f\ddot{\psi}.$$

We will also assume the applied work forces of an internal character and consider the oscillations of machine system with one degree of freedom. Then, substituting for $\lambda\frac{\partial F}{\partial\psi}$, we have

$$\Sigma I\ddot{\phi}+\Sigma m\ddot{y}x-\Sigma m\ddot{x}y+I_f\ddot{\psi}+c\psi=0,$$

which is the equation of oscillation for the total system.

Substituting for the total accelerations in terms of the relative velocity ratios with respect to the relative coördinate and the acceleration of the frame $\ddot{\psi}$, we have

$$\left[\Sigma IK_{\phi\alpha}+\Sigma M(xK_{y\alpha}-yK_{x\alpha})\right]\ddot{\alpha}+\left[\Sigma I\frac{\partial K_{\phi\alpha}}{\partial\alpha}+\Sigma m\left(x\frac{\partial K_{y\alpha}}{\partial\alpha}-y\frac{\partial K_{x\alpha}}{\partial\alpha}\right)\right]\dot{\alpha}^2+\left[I+I_f+\Sigma m(x^2+y^2)\right]\ddot{\psi}+c\psi=0,$$

which, with the generalized equation of motion,

$$\left[\Sigma IK_{\phi\alpha}{}^2+\Sigma m(K_{x\alpha}{}^2+K_{y\alpha}{}^2)\right]\ddot{\alpha}+\left[\Sigma IK_{\phi\alpha}\frac{\partial K_{\phi\alpha}}{\partial\alpha}+\Sigma m\left(K_{x\alpha}\frac{\partial K_{x\alpha}}{\partial\alpha}+K_{y\alpha}\frac{\partial K_{y\alpha}}{\partial\alpha}\right)\right]\dot{\alpha}^2-\Sigma m(yK_{x\alpha}-xK_{y\alpha})\ddot{\psi}=\Sigma(XK_{x\alpha}+YK_{y\alpha}+LK_{\phi\alpha}),$$

determine the oscillation of the system.

(To be continued)

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

PROGRESS ON INTERNATIONAL STANDARDS OF LIGHT AND ELECTRICITY.

For several years negotiations have been in progress toward new international agreements on units and standards for electrical measurements. The latest step was a meeting of an international Advisory Committee on Electricity held in Paris, June 23, 24, and 25, 1930. The minutes of this meeting have just been published so that an authoritative statement can be made regarding its conclusions.

The Advisory Committee on Electricity is a subsidiary of the International Committee on Weights and Measures, having a membership limited to ten. Eight members have been appointed, including one representative each from the national laboratories of Germany, Great Britain, Japan, Soviet Republics, and the United States, and one from the Laboratoire Central d'Electricite at Paris. The other two members appointed are the director of the International Bureau of Weights and Measures and Prof. L. Lombardi of the Royal School of Engineers at Rome.

The advisory committee met first in 1928 when it adopted resolutions looking definitely toward the establishment of electrical units based directly upon the centimeter-gram-second system instead of upon arbitrary standards such as the mercury ohm tube and silver voltameter which are the basis of the present international units.

In 1929 the International Committee on Weights and Measures approved these proposals of the advisory committee and asked for continued assistance on electrical units and standards. It also decided to take up the problem of obtaining international agreement on measurements of light, and instructed the same advisory committee to consider methods, units, and standards for such measurements. Consequently the 1930 meeting of the advisory committee was devoted largely to this new problem.

* Communicated by the Director.

A considerable part of the proceedings centered around documents submitted by the Director of the Bureau of Standards entitled "Photometric Units and Methods," and "Proposals Concerning the Primary Standard of Light." In these documents it was proposed that a "black-body" radiator at the freezing point of pure platinum be adopted as the primary standard of light. This type of standard was originally suggested by Waidner and Burgess in 1908, but has only recently reached the stage of actual realization in a satisfactory form through work which has been carried out in the laboratories of the bureau (See Technical News Bulletin No. 151, p. 107; November, 1929).

The conclusions reached by the advisory committee with regard to standards of light were expressed in the following resolutions (translated from the official French text):

(*A*) Primary standard of light.

(1) The Advisory Committee on Electricity supports the opinion of the International Commission on Illumination that a black-body radiator operated under specified conditions should be adopted, at the present stage of technical development, as the primary standard of light.

(2) The advisory committee requests the several national laboratories to examine the specifications for the construction and operation of the black-body standard submitted by the Bureau of Standards, and to give their opinions regarding the practicability of adopting the standard thus specified.

(3) The advisory committee recommends that additional determinations of the brightness of the black-body radiator be made, especially under the conditions prescribed in the proposed specifications or at least under conditions comparable with those prescribed.

(*B*) Units of light:

(1) The advisory committee believes that it would not be expedient to change the unit which has been in common use in several countries since 1909,

and which has been adopted by the International Commission on Illumination.

(2) The advisory committee expresses the hope that all countries may accept this unit as soon as pending questions (regarding photometric measurements) are settled.

(*C*) Maintenance of units:

(1) The advisory committee proposes that the national laboratories immediately exchange and compare groups of carbon-filament lamps, or of others operated at a corresponding color, and submit for the International Committee on Weights and Measures precise information regarding the relative values of the units of candlepower as now maintained in the several laboratories.

(2) Assuming that any differences disclosed by such comparisons will be removed by agreement among the national laboratories, the advisory committee proposes to make periodic comparisons in the future under the auspices of the international committee.

(3) Recognizing the importance of uniformity and continuity in the values of the practical secondary standards, the advisory committee recommends that any future adjustments of these values which may become necessary as a result of reference to the primary standard shall be made by international agreement approved by the International Committee on Weights and Measures.

(*D*) Secondary standards for diverse colors of light:

(1) The advisory committee recognizes the importance of the coöperative studies now being made by the national laboratories on the measurement of transmission of colored filters, and expresses the hope that these studies will lead to agreement on a standard method for making such measurements as well as to accepted values for the particular filters.

(2) The advisory committee will be pleased to receive for its own information, and for transmission to the International Committee on Weights and Measures, any reports on the progress of these studies which can properly be published.

With regard to electrical units and standards, the following conclusions were reached:

1. With regard to the unit of resistance, the ohm, considering that methods of determining the absolute ohm are sufficiently advanced and that the agreement between the measurements of the coils (secondary standards) of the different laboratories remains within the limits of precision of the measurements, it is not necessary at present to undertake further comparisons of the resistance coils with mercury ohms.

2. As to the units of electromotive force, on the contrary, the international comparisons of standard cells show differences, exceeding the limits of possible precision, between the values of the electromotive force of the standards of different countries. The committee considers that it is absolutely necessary to make new determinations of the electromotive force of the international Weston cells in each national laboratory by means of the silver voltameter.

The conditions under which the silver voltameter should be used are specified sufficiently in the report of the international committee which met in Washington in 1910. The advisory committee recommends, however, that the national laboratories use the Smith or the Kohlrausch voltameter, avoiding organic material, which is detrimental to the electrolytic deposit.

The groups of Weston cells prepared in the several laboratories can be considered as sufficiently constant until the absolute unit of current shall be established.

The advisory committee also made recommendations regarding the equipment of the International Bureau of Weights and Measures and plans for coöperation between that bureau and the national laboratories in work on electric and photometric standards. Since an international General Conference on Weights and Measures is to be held in 1933, the advisory committee decided that it should meet again before that time to study the comparisons of standards which shall

have been made and to assign values in absolute units for the standards of resistance and of electromotive force.

AERONAUTIC RADIO DEVELOPMENTS.

Progress can be reported in the development of a combined transmitter for the simultaneous broadcast of radio telephone and visual type radio range beacon signals. This is designed to remove the principal limitation on the present radio aids to air navigation, viz., that the pilot receives no beacon service while receiving weather or other telephonic information. A master oscillator controls simultaneously a radio telephone transmitter and a beacon transmitter. The former supplies the carrier-frequency waves and the speech-modulated waves to an open antenna. The latter supplies only the radio beacon sidebands to the beacon loop antennas. Filter arrangements and automatic volume control have been worked out to facilitate the reception of these signals aboard the aircraft. The fundamental adjustments necessary in the experimental model of this transmitter have been completed, and performance tests begun at various distances from the station. A number of tests were made approximately 125 miles from the transmitting station; in these tests successful reception of both the voice and beacon signals was accomplished, with indications that this reception could be obtained over greater distances. Satisfactory sharpness of beacon courses was obtained. As a result of these and other tests the feasibility of the combined service is assured.

For some months a detailed study has been in progress on the characteristics and performance of airplane receiving antennas of various types. One object of this investigation was to find an antenna arrangement having the same desirable electrical characteristics as the vertical pole antenna but free from the problems of mechanical vibration and ice formation encountered in the use of the pole antenna. One of the advantages of the vertical pole antenna is its freedom from course errors in radio range beacon reception. A number of different antenna arrangements have been studied, both by theoretical analysis and by practical observation in the air and on the ground. For each antenna studied, the tests in

the air included observation of the received voltage, the localizing effect or variation of the received voltage in the immediate vicinity of the beacon tower, and the course errors as observed by circling the beacon. These were compared directly with results obtained using the vertical pole antenna. The antenna arrangements studied included the inclined antenna, with both forward or backward inclination (one example of the latter being the trailing wire antenna); the horizontal dipole antenna; the horizontal V antenna; the horizontal L antenna; the inclined V antenna; the symmetrical transverse T antenna; and the symmetrical longitudinal T antenna. The symmetrical longitudinal antenna with a vertical lead-in was found to have an advantageous combination of the desired operating characteristics. The two flat top elements lie along a line parallel to and directly above the axis of the fuselage, held by short vertical supports, considerably shorter than the usual pole antenna (10 to 18 inches instead of 5 to 6 feet). Equivalent effective height is secured through the use of the flat top. It is essential that the longitudinal T-antenna be located in a position such that the electrical effect of the airplane frame acting as the counterpoise is symmetrical; this is not always possible in the case of open cockpit airplanes, but is usually possible in cabin airplanes. This type of antenna is free from course errors in radio range beacon reception. It is superior to the vertical pole antenna structurally, and in respect to ice formation, mechanical vibration, and aerodynamic resistance.

The Bureau has begun preliminary work on the development of a radio system to aid in preventing collisions between airplanes. The aim is to give automatic warning to an airplane of the presence and approximate position of any other airplane within a radius of about 3 miles. A fundamental limitation is that only one frequency, or at most a very few frequencies, can be made available for this service. The system involves the continuous transmission of ultra-high-frequency radio waves from each airplane. Directivity of reception or transmission, or both, will inform the pilot of the direction of danger.

INSTRUMENTS FOR MEASURING REFRACTIVE INDEX OF OPTICAL GLASS.

The supposed necessity for using large prisms and telescopes when making accurate measurements on the index of refraction of optical glass, has been investigated recently in the optical instruments section of the bureau, and it was found that a 60° prism with edges measuring three-eights of an inch in length is sufficiently large for use with the most accurate apparatus now available. This work required a determination of the accuracy which is possible in pointing a telescope at a suitable target and also of the way in which this accuracy may vary as larger telescope lenses are used. Another matter depending on prism size is the error made in properly orienting the prism around a vertical axis when measuring its refractive properties. This difficulty is shown to be less important than generally has been supposed, and a satisfactory method of correcting for such small errors is suggested. It is concluded that large telescopes and special methods for correctly orienting the prism are unnecessary in the most accurate measurements of this kind. As a result, small prisms may be used with confidence when testing optical glass for those small but harmful variations in optical density which may be found within a sample intended for use in constructing an optical instrument of high precision.

This investigation will be discussed at greater length in the January number of the *Bureau of Standards Journal of Research*.

SOLARIZATION AND REJUVENATION OF WINDOW GLASS.

In previous reports attention was called to the rejuvenation in ultra-violet transmission of special window glasses on exposure to the sun after exposure to the mercury arc, which emits short wave-length ultra-violet rays not present in sunlight.

During the past summer there has been observed a similar phenomenon of rejuvenation in ultra-violet transmission on exposure of the glass to sunshine filtered through a blue green glass that is opaque to wave-lengths shorter than 340 millimicrons, after solarization by exposure to full sunlight. This

solarization and rejuvenation have been carried through several cycles.

Since solar rays of wave-lengths shorter than about 305 millimicrons are absent in winter sunlight, it may be expected that the solarization caused in summer sunlight will be partly rejuvenated by exposure to winter sunlight. The likelihood of such an occurrence has been under investigation for three seasons, and while the data thus far obtained indicate a possible slight increase in transmission in winter, it amounts to less than 1 per cent., which is barely larger than the errors of observation and requires an averaging of data on more than 2 score samples. From this it is evident that the possibility of recovery in transmission during the winter will be of no use in sales promotion; particularly so when considered in connection with the fact that the wave-lengths (at 302 and lower) at which this increase in transmission may occur, are absent in winter sunlight.

ULTRA-VIOLET REFLECTING POWER OF HYB-LUM.

"Hyb-lum" is a recently marketed alloy of aluminum, in which the alloying elements, amounting to about 2 per cent., are mainly nickel and metals of the chromium group. It is being used in reflectors of therapeutic lamps. Hence, the question arose as to its reflectivity of ultra-violet rays.

The samples examined had a good polish. The reflecting power in the short wave-length ultra-violet, at 300 millimicrons, was found to be about 55 per cent.—which is about 10 per cent. higher than aluminum—gradually increasing to about 75 per cent. in the visible spectrum (550 millimicrons) which is the same as aluminum.

HEAT OF FORMATION OF WATER.

One of the primary functions of the Bureau of Standards is the determination of the standard constants. The heat of formation of water is one of the most important constants in thermochemistry because it is involved directly in the calculation of the heats of formation of practically all organic and of many inorganic compounds.

The usually accepted value for the heat of formation of water is based upon measurements made by Thomsen in 1873, Schuller and Wartha in 1877, and Mixter in 1903. The most reliable of these data are those of Schuller and Wartha, whose average value has an uncertainty of some 8 parts in 10,000, practically all of which lies in the calibration factor of their calorimeter.

The procedure employed in a recent investigation at the bureau was to determine directly the quantitative correspondence between the energy liberated by (1) the reaction of hydrogen and oxygen to form a weighed mass of liquid water and (2) a measured quantity of electrical energy, using the calorimeter as the absorber of the two quantities of energy and its temperature rise as the comparator. In so far as systematic errors are concerned, the absolute accuracy of the result obtained by this substitution method depends principally upon the determination of the mass of water formed, in grams, and of the quantity of electrical energy, in terms of the mean solar second and the international volt and international ohm as maintained at the Bureau of Standards. High precision was obtained by the use of proper calorimetric technic, a sensitive potentiometric system for measuring the electrical power input, a precise timing device, and a suitable balance for weighing the water formed.

The data of two sets, each including the results of nine experiments, give for the heat of formation of liquid water, at 25° C. and a constant pressure of 1 atmosphere, 285,775 international joules per mole (18.0156 grams). The estimated uncertainty in this value is ±40 joules. With the factors 1.0004 and $\frac{1.0004}{4.185}$ this value is equivalent to 285,890 absolute joules and to 68,313 g-cal$_{15}$. The maximum and the average deviations of the experiments, in per cent., are, respectively: 0.031 and 0.019 in Set I; 0.024 and 0.010 in Set II.

The data of Schuller and Wartha, Thomsen, and Mixter, recomputed in terms of the international joule, are in substantial agreement with the value obtained in the present investigation.

This work will form the subject of a report in the January, 1931 number of the *Bureau of Standards Journal of Research.*

DETERMINATION OF MAGNESIUM IN PORTLAND CEMENT.

The maximum amount of magnesia (MgO) allowed by the Federal Specification 1a and the Standard Specification for Portland Cement (C9–26) of the American Society for Testing Materials, is 5.00 per cent. (plus a tolerance of 0.4 per cent.). Since a great deal of cement is purchased under these specifications it follows that many determinations of magnesia are required. At present the phosphate method is standard, and is quite accurate and proper for umpire analyses, but it is rather lengthy. The precipitation of magnesium by the reagent 8-hydroxyquinoline has been studied at the bureau, and in the January number of its *Journal of Research* a method is recommended for determining magnesium by use of this reagent. The recommended procedure is accurate and much more rapid than the standard phosphate method.

EXTRACTION OF BITUMINOUS SATURATED FELTS.

Federal specifications for bituminous saturated felt for roofing and waterproofing purposes specify the weight of the moisture-free desaturated felt per 100 square feet, and require a piece of felt two inches wide and the full width of the roll (32 or 36 inches) to be desaturated for this determination.

For several months the bureau has used Soxhlet-type extractors for this purpose with very satisfactory results. The extraction tube of the apparatus used is of sufficient length to accommodate two samples rolled, and glass beads are added with the samples to reduce the volume of solvent, thus securing more frequent extractions per unit time. The vapor tube is insulated by wrapping with asbestos cord and the vapors are condensed in a water-cooled, worm condenser. Heat is supplied by a small electric hot plate and extractions are complete after several hours. For felts carrying a large percentage of bitumen the solvent should be changed after several extractions to prevent foaming.

This method is economical of both time and solvent, it is in accordance with the specification directions, and has the advantage of not exposing the operator to solvent fumes during the extraction.

EFFECT OF VARIATIONS IN COMPOSITION ON PROPERTIES OF VITREOUS ENAMELS.

In an item which appeared on page 78 of Technical News Bulletin No. 160 (August, 1930) it was reported that enamels having identical calculated chemical compositions, but prepared from different raw mixes, had shown a distinct difference in softening temperature. It was further stated that any final conclusions concerning this observation should be withheld until the results of chemical analyses of the frits were available. When those results became available, differences in composition were shown to exist which would account for a difference in fusibility in the observed direction.

Repetitions of the experiment, which have been carried out since that time, confirm the observation that appreciable differences in fusibility occur when the enamels are prepared in refractory crucibles and are agitated during smelting by continuous rotation of the crucibles. This is the method by which the enamels were initially prepared, of which the behavior was previously reported. When smelted in stationary refractory crucibles with occasional stirring, the differences between enamels were less noticeable.

When the smelting was carried out in platinum crucibles, no appreciable effect of variations in raw materials, to give the same calculated melted composition, was observed by means of cone deformation tests of "fusion block" tests. (Fusion blocks contain compartments for packing in a definite volume of powdered test material, which softens upon elevation of the temperature and flows down a steeply inclined plane bearing graduation marks. Under this test some enamels flow at considerably lower temperatures than others.) The batch mixes which were used to obtain the described results are given below. Each raw material was analyzed, and on the basis of the results of analysis the batches were calculated to give as near as possible the same melted composition.

Material.	Batch Mixes.	Parts by Weight.
Borax	43.00	19.23
Feldspar	30.30	30.30
Quartz	29.76	.....
Sodium carbonate	8.92	.....
Sodium nitrate	5.00	5.00
Sodium silicate	.....	39.19
Boric acid	.....	15.67
Fluorspar	8.06	8.06
Cobalt oxide	0.60	0.60
Manganese oxide	.194	1.94

The object of these tests was merely to determine whether or not any correlations which might be established between the compositions and the properties of typical ground coat enamels now being studied would be vitiated by a radical variation in the raw batch used to obtain any given melted composition. It was incidentally noted that batches melted in the rotating refractory crucibles had somewhat elevated softening temperatures as compared to those melted in platinum.

DESTRUCTIVE WEATHERING OF ROOFING SLATE.

How a natural rock composed mainly of inert minerals and possessing high strength as well as very low porosity is attacked by the weather is a problem of considerable interest to both producers and users of roofing slate. While slate roofs are commonly considered as among the most permanent types, there are instances where this material has not come up to expectations.

During the past year the Bureau of Standards has studied this question with the particular object of devising specifications tests. The study of weathered slates from roofs, together with numerous tests, have indicated that the decay is due to a combination of chemical and physical processes. Frost action apparently takes very little part in the decay until the material has been considerably altered from its original condition by certain chemical transformations.

It is a well known fact that decayed slate usually shows the presence of quite appreciable amounts of gypsum, although the fresh slate is practically free from this substance. Some

investigators have accounted for the formation of gypsum in decayed slate by assuming that the small amounts of sulphuric acid in the air react with the calcitic impurities. This may account for a part of the gypsum but it has been proved that this substance can also be formed in some slates by merely soaking and drying them several times. The formation of gypsum during the weathering process affords a clue to the cause of decay and indicates that the mineral impurities concerned in the reaction are calcite and pyrite, the necessary oxygen to complete the sulphate radicle being drawn from the air. Apparently free carbon which is usually present in slate aids the reaction. The conversion of calcite to gypsum brings about expansive forces within the slate since the gypsum molecule requires more space than the calcite molecule. Scaling of the surface of the slate results, and this action is practically always more pronounced on the covered portion of the shingle than on the part exposed to the weather. This peculiar condition is probably caused by water penetrating between the shingles during rains and the slower rate of drying there than on the exposed surface. Laboratory experiments indicate that the formation of gypsum within the slate does not occur when the material is continually wet and that periods of moist conditions alternating with periods of dry conditions are necessary to cause the reaction. Slates having only a small amount of calcitic impurity are affected by this type of decay at a very slow rate. A very dense slate containing considerable calcite is not decomposed as rapidly as a more porous one with a similar calcite content. Some of our domestic roofing slates may be depended upon for more than 30 years of serviceable results while others are apparently good for more than 100 years.

Resistance to Flow of Different Parts of the Apparatus Used in Producing High Vacua. W. KLOSE. (*Physik. Zeit.*, May 15, 1930.) While the flow of rarefied gas through capillary tubes has been investigated but slight attention has been paid to the influence of the channel form in a bent tube, yet just such information as this is needed in the design of apparatus for producing high vacua.

By McLeod gauges the pressures at the two ends of the tube under investigation were measured. The difference of these gave the driving pressure and their average the mean pressure in the tube. The mass of air flowing through in a given time per unit difference of pressure was determined. The average pressures ranged from 10×10^{-4} to 200×10^{-4} mm. of mercury. The results of observation reduced to equivalent lengths and diameters of tube are made into curves, on each of which appears the data for a straight tube. There is scarcely any difference between the curve for this simple form and those derived for a tube with four right-angled bends, for one with four T-shaped enlargements or for a wide cock. On the contrary a resistance greater by 15 or 30 per cent. was found for a narrow cock and for a cooling arrangement. "In the field of mean pressures ranging from 10 to 200 ten-thousandths mm. of mercury the shape of channel is without essential influence upon the velocity of flow, provided wide tubes are used. But constrictions, even though short, reduce it considerably." In a tube of radius .695 cm. and length 100 cm., bent so as to have four right angles, an increase of pressure from 10^{-4} mm. to 201^{-4} mm. mercury, that is twenty fold, caused only about 60 per cent. increase in the rate of flow.

G. F. S.

The Splitting Strength of Mica. J. W. OBREIMOFF. (*Proc. Roy. Soc.*, A 805.) He finds that mica split in air has a surface energy of 1500 ergs per cm.^{2} Two newly split mica surfaces stick together even without the application of any pressure. When particles of dust, scratches or other interfering circumstances do not prevent the two surfaces from approaching closely the mica is completely restored. For mica split *in vacuo* the surface energy is 20,000 ergs/cm.^{2}

In a vacuum the operation of splitting makes the mica slightly luminescent. "After splitting mica in high vacuum, the inner surfaces of the glass vessel and of the mica itself are covered with electrical charges.

G. F. S.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING DECEMBER, 17, 1930.

The President, Mr. Nathan Hayward, called the regular monthly meeting of The Franklin Institute to order at eight-fifteen p.m. He announced that it was the meeting at which certain business had to be transacted and called upon the Secretary of the Institute for a report.

The Secretary announced that the minutes of the November meeting had been printed in full in the December number of the Journal of The Franklin Institute and moved that those minutes be approved as printed. The President asked if there were objections and hearing none announced the minutes approved as printed.

The Secretary announced that since the last membership report nine Resident members and two Non-resident members had been added to the roll.

He announced further that at this meeting nominations must be made for the election of officers and members of the Board of Managers which would be held at the annual meeting of the Institute in January. He stated that all nominations must be presented in writing, must be signed by at least two members of the Institute, and must bear the statement that the candidates if elected would fulfil the duties of their offices. He pointed out that there were several nominations to fill vacancies in the Board of Managers which had unfortunately been caused by death or resignation. Under the By-laws these vacancies are filled by appointment until the ensuing annual meeting, at which time they must be filled by election, for the unexpired terms.

The following nominations were proposed:

To serve one year:		*Nominated by*
President	Mr. Nathan Hayward	Mr. Henry Howson
		Mr. Walton Forstall
		Mr. W. Chattin Wetherill
Vice-Presidents	Mr. Henry Howson	
	Mr. C. C. Tutwiler	Mr. R. W. Lesley
	Mr. Walton Forstall	Mr. Haseltine Smith
	Mr. W. Chattin Wetherill	Mr. Alfred Rigling
Treasurer	Mr. Benjamin Franklin	Dr. G. A. Hoadley
		Mr. J. S. Rogers
To serve three years:		
Board of Managers	Mr. F. T. Chambers	
	Mr. C. H. K. Curtis	
	Mr. S. S. Fels	
	Mr. M. Lloyd, Jr.	
	Mr. R. W. Lesley	
	Mr. M. S. Morgan	Dr. A. W. Goodspeed
	Mr. E. H. Sanborn	Mr. H. B. Allen
	Mr. S. T. Wagner	Dr. H. C. Porter
To serve two years:		
Board of Managers	Mr. A. Atwater Kent	
To serve one year:		
Board of Managers	Mr. L. H. Kinnard	
	Mr. R. McLean	

After the Secretary had presented the above nominations, the President called for nominations from the floor. None were offered and the President declared the nominations closed.

Dr. McClenahan then made a statement concerning the Christmas Week Lectures. The subject will be "The Scientific Experiments of Benjamin Franklin." The lecturer will be Dr. E. E. Free of New York University, a noted expositor of scientific things. The dates will be December twenty-ninth, thirtieth and thirty-first, at three p.m. He recited a statement which had been made by Dr. Free, to the effect that the kite experiment had been successfully performed in his laboratory so that at the Christmas Week Lectures the upper atmosphere of the room would be electrified, a kite would be sent up, and electrical charges be drawn down just as Franklin did. The Secretary expressed the belief that, judging from the outline, Dr. Free's lectures would be of the highest quality and very entertaining as well as instructive. He stated that of the two hundred and seventy-five seats available in the room, one hundred and forty-seven had already been disposed of. He suggested the propriety of presenting a ticket of admission to any scientifically, curious minded young person.

There being no further business, the President of the Institute presented the lecturer of the evening, Dr. Saul Dushman, Assistant Director of the Research Laboratory of the General Electric Company at Schenectady, who spoke in a most delightful fashion and with full authority on "Methods of Producing High Vacua." Dr. Dushman presented a review of the development of processes for the production of extremely high vacua, including the labors of his co-workers at Schenectady as well as of research men in other parts of the world.

The paper was followed by a brisk discussion from the floor. The meeting adjourned at ten-one p.m., with an expression by the President of high appreciation of Dr. Dushman's talk.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Thursday, December 11, 1930.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 11, 1930.

MR. HENRY B. ALLEN *in the Chair.*

The following reports were presented for first reading:

No. 2856: Vapor Phase Process of Treating Petroleum Hydrocarbons.

No. 2932, No. 2933 and No. 2934 } Franklin Medal.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, December 17, 1930.)

RESIDENT.

DR. LESLIE R. BACON, Chemist, American Doucil Company, 121 South Third Street, Philadelphia, Pa. For mailing: 536 Mohawk Avenue, Norwood, Pa.

DR. LAWRENCE M. HENDERSON, Supervising Research Chemist, The Atlantic Refining Company, 3144 Passyunk Avenue, Philadelphia, Pa. For mailing: 211 Dudley Avenue, Narberth, Pa.

MR. WILLIAM J. JEFFRIES, Metallurgical Chemist, Walnut Park Plaza, Sixty-third and Walnut Streets, Philadelphia, Pa.

MR. GEORGE F. PETTINOS, Manufacturer and Miner, 1206 Locust Street, Philadelphia, Pa. For mailing: 739 Beacon Lane, Merion, Pa.

MR. KENNETH E. STUART, Mechanical and Electrical Engineer, Hooker Electrochemical Company, Niagara Falls, New York. For mailing: 367 Baird Road, Merion, Pa.

MR. SIGURD WESTBERG, Metallurgical Engineer, 4800 Walnut Street, Philadelphia, Pa.

NON-RESIDENT.

MR. D. E. MOAT, Leeds and Northrup Company, 1942 Union Trust Building, Cleveland, Ohio.

CHANGES OF ADDRESS.

MR. CARL G. BARTH, 472 Whitney Avenue, New Haven, Conn.

MR. A. W. K. BILLINGS, c/o Canadian and General Finance Co., Ltd., 25 King Street, West, Toronto, Ontario, Canada.

MR. W. A. CONVERSE, 7023 Constance Avenue, Chicago, Illinois.

MR. J. C. DACOSTA, 3D, c/o Dr. Roger J. Hutchinson, Causeway Side, Haslemere, Surrey, England.

MR. CHESTER L. DAVIS, 1227 National Press Building, Washington, D. C.

MR. MILTON W. DEISLEY, 222 South Queen Street, Lancaster, Pa.

MR. EDWIN D. A. FRANK, 1714 North Prospect Avenue, Apt. 2, Milwaukee, Wisconsin.

MR. HAROLD GOODWIN, JR., 313 Maple Avenue, Wyncote, Pa.

MR. WALTER F. GRAHAM, Technical Director, Caskey Brass and Bronze Works, Inc., Richmond and York Streets, Philadelphia, Pa.

MR. J. HENRY HALLBERG, 303 Fourth Avenue, New York City, N. Y.

LIEUT. S. LEONARD KENT, JR., 630 Winsford Road, Bryn Mawr, Pa.

MR. WILLIAM PLASS, Bellerick Apartments, 301 South Fifteenth Street, Philadelphia, Pa.

MR. S. B. RICKERSBERG, Gwynedd, Pa.

MR. AUGUSTUS SMITH, Roselle, N. J.

MR. PAUL E. TODD, 619 Thirteenth Avenue, S.E., Minneapolis, Minn.

MR. JOHN H. WIDMYER, 216 East King Street, Lancaster, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

ADAM, ALASTAIR THOMAS. Wire-Drawing and the Cold Working of Steel. 1925.

AITKEN, ROBERT. Ultra-Violet Radiations and Their Uses. 1930.

American Foundrymen's Association. Index of the Transactions. Volumes 30–37. 1930.

Automatic Signal Corporation. General Traffic Survey with Recommendations for Electro-Matic Control, Prepared for the City of Providence, R. I. 1929.

BARKER, T. V. The Study of Crystals: a General Introduction. 1930.

BARNARD, GEORGE P. The Selenium Cell: Its Properties and Applications. No date.

BARTLETT, A. C. The Theory of Electrical Artificial Lines and Filters. 1930.

BOYD, JAMES E. Mechanics: a Text-Book for Engineers. Second edition. 1930.

Conference Internationale des Grands Réseaux Électriques à Haute Tension. Compte-Rendu des Travaux de la Cinquième Session. Tomes 1–3. 1929.

DUNCAN, JOHN CHARLES. Astronomy: a Text Book. Second edition, revised. 1930.

FULMER, ELLIS I., AND C. H. WERKMAN. An Index to the Chemical Action of Microorganisms on the Non-Nitrogenous Organic Compounds. 1930.

GILMORE, A. B. Manual de la Industria Azucarera Cubana: the Cuban Sugar Manual. 1928.

GILMORE, A. B. Porto Rico Sugar Manual Including Data on Santo Domingo Mills. 1930.

GREGORY, J. W. The Rift Valleys and Geology of East Africa. 1921.

GROESBECK, HARRY A., JR. Practical Photo-Engraving Explained in Fifty Letters from a Photo-Engraver to his Son. 1930.

HASKELL, ALLAN C. Graphic Charts in Business. Third edition. 1928.

HAUSER, ERNST A. Latex: Its Occurrence, Collection, Properties and Technical Applications. With a Patent Review compiled by Carl Boehm von Boernegg. 1930.

HENRICI, ARTHUR T. Molds, Yeasts, and Actinomycetes: a Handbook for Students of Bacteriology. 1930.

JEANS, SIR JAMES. The Mysterious Universe. 1930.

JONES, FRANKLIN D., Editor. Die Design and Diemaking Practice. 1930.

Kaiser-Wilhelm-Institut für Kohlenforschung in Mülheim-Ruhr. Gesammelte Abhandlungen zur Kenntnis der Kohle. Neunter Band. 1930.

KIEFER, PAUL J., AND MILTON C. STEWART. Principles of Engineering Thermodynamics. 1930.

LIDDELL, DONALD M. The Metallurgists' and Chemists' Handbook. Third edition, revised and enlarged. 1930.

LOCKETT, ARTHUR. Camera Lenses. 1928.

LUCKIESH, M. Artificial Sunlight: Combining Radiation for Health with Light for Vision. 1930.

NEWBURGH, L. H., AND MARGARET W. JOHNSTON. The Exchange of Energy Between Man and the Environment. 1930.

OSGOOD, WENTWORTH H. Increasing the Recovery of Petroleum. First edition. Two volumes. 1930.

PREGL, FRITZ. Quantitative Organic Microanalysis. Second English edition, translated from the third revised and enlarged German edition by Ernest Fyleman. 1930.

Royal Society of London. Philosophical Transactions. Series B. Volume 218. 1930.

SAIDLA, LEO E., AND WARREN E. GIBBS. Science and the Scientific Mind. First edition. 1930.

SCHWAMB, PETER, ALLYNE L. MERRILL, AND WALTER H. JAMES. Elements of Mechanism. Fourth edition. 1930.

SMITH, WILLIAM GRISWOLD. Engineering Kinematics. Second edition. 1930.

STEELL, WILLIS. Benjamin Franklin of Paris, 1776–1785. 1928.

STEWART, ALFRED W. Recent Advances in Physical and Inorganic Chemistry. Sixth edition. 1930.

STEWART, C. J. Aircraft Instruments. 1930.

TERRILL, H. M., AND C. T. ULREY. X-Ray Technology. The Production, Measurement, and Applications of X-Rays. 1930.

TIMM, JOHN ARREND. An Introduction to Chemistry: a Pandemic Text. First edition. 1930.

UNDERWOOD, G. Estimating Construction Costs. First edition. 1930.

URQUHART, LEONARD CHURCH, AND CHARLES EDWARD O'ROURKE. Design of Steel Structures. First edition. 1930.

WALKER, J. H., AND SABIN CROCKER. Piping Handbook. First edition. 1930.

WASHINGTON, HENRY S. The Chemical Analysis of Rocks. Fourth edition, rewritten and enlarged. 1930.

WHEELER, E. The Manufacture of Artificial Silk with Special Reference to the Viscose Process. 1928.

BOOK REVIEWS.

ENGLISH-GERMAN AND GERMAN-ENGLISH DICTIONARY OF CHEMISTRY. By Willy H. Thurow, Member of The Franklin Institute, Philadelphia, etc. Part I. English-German. VII–261 pages. Berlin, Verlag Dr. Arthur Tetzlaff, 1929.

In an American scientific library this volume should hold a rather unique position. We are accustomed to see on the reference shelves numerous German-English dictionaries but seldom if ever a scientific compendium of the reverse type. Nevertheless, in this day of foreign branch offices or subsidiary companies, increased traveling conveniences promote personal interviews and subsequent correspondence. American research and technical men quite frequently write to their German associates and acquaintances in the language of the one addressed not only as an expediency but for courtesy as well. For such writers this book should prove its worth.

For the purpose of conciseness the author has given first place to the more general meanings and interpretations of the words. Thus the book should find very extensive application in general scientific fields. The author contemplates a later edition with a guide to pronunciation should present users of the book think it desirable.

T. K. CLEVELAND

SCIENCE AND THE SCIENTIFIC MIND. By Leo E. Saidla, Assistant Professor of English, Polytechnic Institute of Brooklyn, and Warren E. Gibbs, Instructor in English, Columbia University, and lecturer in English, Polytechnic Institute of Brooklyn, Evening Division. xiv–506 pages, First Edition, 12mo. New York, McGraw-Hill Book Company, Inc., 1930. Price $3.00.

A collection of non-technical scientific articles of merit primarily intended to supplement an advanced course in composition for students of science and technology. Its main theme is to define and illustrate the scientific mental attitude or the scientific habit of thought. As a textbook it far surpasses the limitations imposed by popular definition. It has more the nature of an anthology of the scientific mind which is presented both subjectively and objectively—by the man who has one or, if you will, who is possessed by it and, by the man from whom it is completely detached. In addition, we are given the opportunity to observe what an integral part science has played, is playing and supposedly will play in the mental, moral, social and physical phases of human existence.

The list of authors includes many outstanding leaders in the "sciences" and "humanities." Among scientists of the past we find Tnydall and Huxley twice represented; of those contemporary, Robert Andrews Millikan, Edwin E. Slosson, Henry Fairfield Osborn, Frederic Soddy, Bertrand Russell, J. S. Haldane and Michael Pupin. Additional essays by men well known in their particular spheres of learning are included for the purpose of filling in and rounding out the whole expository structure.

The subject matter has been conveniently grouped under six headings which include essays on (1) Science in General, (2) the Scientific Mind, (3) Scientific Motive, (4) Science and Culture, (5) Science and Civilization and (6) Science and the Future. Although the greater number of essays embraced by these captions have been written by scientists the balance, which have a similar superlative quality, may claim as their creators such men as Truman L. Kelly, psychologist; J. W. Mackail, a foremost scholar in Greek and Latin; George E. Woodberry, poet; Thorstein Veblen, economist and Frederick S. Marvin, historian.

Unlike the human body a book may have more than one appendix and all of them rate an importance beyond vestigial rank. The appendices of this book should possess great value for the student of scientific exposition. They include: *A*, Suggestions for the study of the structure of an essay; *B*, Topics for oral or written discussion and reports; *C*, A very extensive and well classified reading list, and *D*, Numerous biographical notes.

T. K. CLEVELAND.

THE THEORY OF ELECTRICAL ARTIFICIAL LINES AND FILTERS. By A. C. Bartlett, B. A., Member of the Research Staff of the General Electric Co., Ltd. ix–155 pages, 160 illustrations, 8vo. New York, John Wiley & Sons, Inc., 1930. Price $3.50.

The book is written as a general introduction to the theory of artificial transmission lines, line balances, filters and phase shifters. It is intended to prepare the reader for further study of the subject through the literature. The first three chapters are considered to be the most important. These deal with the theory of T and Π sections artificial lines in considerable detail, the general theory

of repeated networks and some generalizations of T and Π sections. The remaining chapters are concerned chiefly with extensions or applications. They include such subjects as Filters, Homographic Transformation, Coil Loaded Telephone Cable and Multistage Amplifier.

The book is quite mathematical in nature and is well supplied with illustrations.

T. K. CLEVELAND.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 359. An Investigation of the Effectiveness of Ignition Sparks. By Melville F. Peters, Wayne L. Summerville and Merlin Davis. 13 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

The effectiveness of ignition sparks was determined by measuring the volume (or mass) of hydrogen and of oxygen which combines at low pressures. The sparks were generated by a magneto and an ignition spark coil. It was found that with constant energy the amount of reaction increases as the capacitance component of the spark increases. The use of a series of spark gap may decrease or increase the amount of reaction, the effect depending upon the amount and the distribution of capacitance in the circuit. So far as the work has progressed, it has been found that sparks reported by other investigators as being most efficient for igniting lean mixtures cause the largest amount of reaction. Differences between the amount of reaction with a magneto spark and an ignition spark coil were noted. The method appears to offer a means of determining the most efficient spark generator for internal-combustion engines as well as determining a relation between the character of spark, energy, and effectiveness in igniting inflammable mixtures.

Report No. 361. Experimental Determination of Jet Boundary Corrections for Airfoil Tests in Four Open Wind Tunnel Jets of Different Shapes. By Montgomery Knight and Thomas A. Harris. 27 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price twenty cents.

The tests reported herein were conducted at the Langley Memorial Aeronautical Laboratory, Langley Field, Va., to determine experimentally the instantaneous pressures at the discharge orifice of a common-rail fuel injection system in which the timing valve and cut-off valve were at some distance from the automatic fuel injection valve, and also to determine the methods by which the pressure fluctuations could be controlled.

The instantaneous pressures at the discharge orifice of a common-rail fuel injection system were determined by analyzing the stem-lift records of an automatic injection valve. The fuel injection was obtained by releasing fuel from a reservoir under high pressure by means of a cam-operated timing valve. The period of injection was controlled by the opening of a cam-operated by-pass valve which reduced the fuel pressure between the timing valve and the injection valve. An injection system of this type assures the same rate of fuel discharge regardless of engine speed. The results show that pressure wave phenomena occur between the high-pressure reservoir and the discharge orifice, but that these pressure

waves can be controlled so as to be advantageous to the injection of the fuel. The results also give data applicable to the design of such an injection system for a high-speed compression-ignition engine.

Report No. 363. Pressure Fluctuations in a Common-Rail Fuel Injection System. By A. M. Rothrock. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1930. Price ten cents.

This experimental investigation was conducted primarily for the purpose of obtaining a method of correcting to free air conditions the results of airfoil force tests in four open wind tunnel jets of different shapes. Tests were also made to determine whether the jet boundaries had any appreciable effect on the pitching moments of a complete airplane model. The investigation was conducted in the Atmospheric Wind Tunnel of the Langley Memorial Aeronautical Laboratory.

The method of obtaining the airfoil corrections utilized the results of force tests made in each jet on three similar monoplane airfoil set-ups of different sizes. The data from the tests in one of the jets which was circular were extrapolated to the condition of infinite air space, and the results were found to agree with those obtained by means of Prandtl's theoretical method of correction. On this basis corrections were then obtained for all the other airfoil tests.

Satisfactory corrections for the effect of the boundaries of the various jets were obtained for all the airfoils tested, the span of the largest being 0.75 of the jet width. The corrections for angle of attack were, in general, larger than those of drag. The boundaries had no appreciable effect on the pitching moments of either the airfoils or the complete airplane model. Increasing turbulence appeared to increase the minimum drag and maximum lift and to decrease the pitching moment. R.

BARLOW'S TABLES OF SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS AND RECIPROCALS OF ALL INTEGER NUMBERS UP TO 10,000. Edited by L. J. Comrie, M.A., Ph.D., Superintendent of His Majesty's Nautical Almanac Office. Third Edition, 208 pages, 8vo. London, E. & F. N. Spon, Ltd., and New York, Spon & Chamberlain, 1930. Price $3.00.

It is quite impossible to give a book like this the ordinary kind of review. Peter Barlow brought out the first edition in 1814, and in 1840 a second edition was edited by Augustus De Morgan. The latter has been in use until now, and has always been a standard work extensively used by computers. Its long continued popularity testifies to its excellence far more eloquently than could any word of the reviewer. By the same token, however, the publication of a third edition is a real event in the history of mathematical tables.

The new editor is a worthy successor to Barlow and De Morgan. Dr. Comrie, after taking his doctorate at Cambridge University, came to the United States. He spent several years at the Sproul Observatory of Swarthmore College, then a year at Dearborn Observatory, Northwestern University. In 1925 he returned to England, to become Deputy Superintendent of the British Nautical Almanac. Last summer, upon the retirement of Dr. P. H. Cowell, he was made Superintendent, greatly to the gratification of his numerous American friends. He has made a particular study of mathematical tables, in which field he is recognized as a leading authority. Of special interest has been his work in

applying calculating machines (many, incidentally, of American manufacture) to the preparation of such tables and to the solution of astronomical and other scientific problems. One of his most spectacular, as well as most valuable, achievements of this kind has been the use of the Hollerith machine for calculating tables of the moon's positions for years to come. This machine has often been used for tabulating census returns in the United States and other countries, but never before has it been applied to an astronomical problem.

In the preparation of the work before us, Dr. Comrie has made use of the Hollerith and other machines. Most valuable of the new features is a table showing the value of $\sqrt{10n}$, thus giving the square root of every tenth number between, 10,000 and 100,000. "This," says Dr. Comrie, "enables the user, when extracting a square root, to have the benefit of a four-figure argument in all cases."

In this table, the values from 2500 to 10,000 were obtained from Hülsse's edition of Vega's *Sammlung Mathematischer Tafeln* by interpolating to tenths the values between 250 and 1000. From 1000 to 2500 the values were obtained by dividing by two every fourth value from 4000 to 10,000. It was in this work that machines were used.

"The initial stages of the interpolation," says the editor, "were done by a Hollerith tabulating machine, and the final stage, consisting of a building up from second differences, with the printing of number, square root, first and second differences, was done on a Class 11 Burroughs adding machine, which lends itself perfectly to the art of constructing tables by integration in the manner contemplated by Babbage when he attempted to build his difference engine."

Another useful innovation is the printing of differences for the tables of square and cube roots and reciprocals, thus aiding the user in interpolation. Fourth powers are given for numbers up to 1000 and all powers up to the tenth for every number up to 100, as well as the eleventh to twentieth powers of the numbers from 1 to 10. These tables were in Barlow's original edition, but were deleted by De Morgan.

Physically, the book is much more attractive than either of the previous editions. The old 1840 stereo plates, used until now, were rather the worse for wear, and, in addition, they were in an old style type, rather small and inconveniently arranged. The new type is much clearer. Though the book is larger, it is much easier to consult. One of the important reasons for the popularity of the work in the past has been the remarkable scarcity of errors. The few errors that Dr. Comrie found have been corrected, so now it comes just about as close as possible to that exceedingly great rarity—a perfect book.

JAMES STOKLEY.

PUBLICATIONS RECEIVED.

Non-Interpolating Logarithms, Cologarithms and Antilogarithms, by Frederick W. Johnson, unpaged, tables, 8vo. San Francisco, The Simplified Series Publishing Company, 1930. Price $2.25.

Cours d'Optique a l'Usage de l'Enseignement Superieur, par G. Bruhat, 756 pages, tables, illustrations, 8vo. Paris, Masson et Cie, 1931. Price 100 Fr.

The Development of Physiological Chemistry in the United States, by Russell H.

Chittenden, 427 pages, 8vo. New York, The Chemical Catalog Company, Inc., 1930. Price $6.00.

Conferences d'Actualites Scientifiques et Industrielles, No. X. Temperature des Flammes, par G. Ribaud, 43 pages, illustrations, tables, 8vo. Price 5 frs. *No. XI. Anisotropie des Molecules*, Effet Raman, par Jean Cabannes, 66 pages, illustrations, 8vo. Price 8 frs. *No. XV. Les Statistiques Quantiques et Leur Application Aux Electrons Libres Dans Les Metaux*, par L. Brillouin, 44 pages, 8vo. Price 5 frs. *No. XVI. La Constitution des Cometes*, par F. Baldet, 23 pages, plates, 8vo. Price 5 frs. *No. XVII. La Structure et les Mouvements de l'Univers Stellaire*, par G. Darmois, 16 pages, 8vo. Paris, Librairie Scientifique Herman et Cie, 1930. Price 3 frs.

Canada, Department of Mines, Mines Branch. Investigations of Fuels and Fuel Testing, 1928. 71 pages, tables, illustrations, plates, 8vo. Ottawa, King's Printer, 1930.

Manuale Pratico di Telegrafia Sottomarina (Cablografia), por Ing. Italo de Giuli, 384 pages, plates, illustrations, 16mo. Milano, Ulrico Hoepli, 1931. Price Lire 30.

A Quantitative Study of Achromatic and Chromatic Sensitivity from Center to Periphery of the Visual Field, by Hazel Austin Wentworth, Psychological Monographs, Volume 40, No. 3, 192 pages, tables, illustrations, 8vo. Princeton, Psychological Review Company, 1930.

Barlow's Tables of Squares, Cubes, Square Roots, and Reciprocals of all integer numbers up to 10,000. Third edition edited by L. J. Compie, M.A., Ph.D. 220 pages, 8vo. London, E. & F. N. Spon, 1930. Price 7 shillings 9 d.

U. S. Commissioner of Lighthouses, Annual Report to the Secretary of Commerce for the Fiscal Year Ended June 30, 1930, 43 pages, tables, 8vo. Washington, Government Printing Office, 1930. Price 5 cents.

U. S. Bureau of Standards, Annual Report of the Director to the Secretary of Commerce for the Fiscal Year Ended June 30, 1930, 53 pages, tables, 8vo. Washington, Government Printing Office, 1930. Price 10 cents.

Yearbook of Railroad Information, 94 pages, illustrations, tables, 8vo. New York, Committee on Public Relations of the Eastern Railroads, 1930.

National Advisory Committee for Aeronautics. Technical notes: No. 354, An Investigation of the Phenomenon of Separation in the Air Flow Around Simple Quadric Cylinders, by John F. Parson and Jarvis A. Wallen, 26 pages, tables, illustrations, plates, quarto, Washington, Committee, 1930. No. 356. Some Characteristics of Fuel Sprays from Open Nozzles, by A. M. Rothrock and D. W. Lee, Langley Memorial Aeronautical Laboratory, 11 pages, illustrations, quarto, Washington, Committee, 1930. No. 357, Bending Tests of Metal Monocoque Fuselage Construction, by Ralph W. Mossman and Russell G. Robinson, 38 pages, illustrations, tables, quarto. Washington, Committee, 1930. No. 358, Experiments with a Model Water Tunnel, by Eastman N. Jacobs and Ira H. Abbott, Langley Memorial Aeronautical Laboratory, 10 pages, illustrations, tables, quarto. Washington, Committee, 1930.

CURRENT TOPICS.

The New Acoustics. W. H. ECCLES. (*Proc. Physical Society*, London, No. 229.) This interesting paper was delivered as the presidential address to the Physical Society. Rayleigh's Theory of Sound, Helmholtz's The Sensations of Tone, and Lamb's Dynamical Theory of Sound are cited as typical of the range of acoustics until new fields were explored a decade ago. In recent years vibrations of very low frequency as well as supersonic vibrations have become the subjects of investigation. Besides this, "the intimate connection with electrical science which has been brought about by the study of the telephone has reacted upon the older and more formal science of acoustics by adding a new technique, new ideas and a new jargon for expressing these new things. The advent of these novelties has been expedited, perhaps even provoked, by the attention given to acoustics during the war; but perhaps the chief single cause of the practical importance now accorded to acoustics is the rise of broadcasting. That is to say, the study of acoustics is now most keenly pursued, not for its intrinsic scientific interest alone, but because of its utilisable results."

New apparatus has been applied to acoustical problems, and much of this is electrical in character as, for instance, the condenser microphone. Now that there are microphones that convert sound vibrations into electrical currents without great distortion it is true that most quantitative acoustical measurements are made upon these resulting electrical currents. The triode amplifier can magnify currents delivering a power as small as 10^{-10} watt and it can magnify by a factor of 10^{11}. By the joint use of a sensitive microphone and of high amplification "it is now possible to detect variations of air pressure of acoustic frequency which are too faint to affect the most sensitive ear." In the reverse direction the triode makes it possible to produce sounds smoothly variable both in intensity and frequency over wide ranges. Electrical filter circuits permit only those currents to pass that have an assignable extent of frequencies. They are used to remove constituent frequencies before the currents are translated into sound vibrations.

Acoustics further profits from the study that has been made of electrical vibrations in circuits and networks. This comes from the fact that all vibrations satisfy about the same linear differential equations. "Further, by adopting the phraseology of the electrician into acoustics, so that translation of the acoustic problem

into the electrical problem becomes automatic, a language for thinking and talking becomes available and is found to clear the mind and to assist reasoning. Moreover, where some new conception occurs in electrical vibration theory, we can immediately seek the analogy in sound." Thus Van der Pol's discussion of a relaxation oscillation in the circuit of a neon tube throws light on the squeak made by a slate pencil running up a slate.

The author now turns to the chief advances in the field of acoustics. In architectural acoustics Sabine of Harvard opened up the entire field by his quantitative investigations on the decay of sound and its absorption. The discovery of the amplifier gave new life to the study of these subjects in several countries. "Accuracy has been introduced into our prescriptions of the best reverberation value for speech, for singing, for instrumental music respectively, and in halls of various sizes. It is now possible to measure the defects of a given auditorium and to calculate with fair certainty the amount of absorbing material that should be permanently introduced to correct these defects. Better still, it is now easy to plan buildings before their erection to have a desired reverberation time—in fact to be of good acoustic quality; and the design of noise-proof houses and laboratories has progressed very rapidly."

Definite data on the functioning of the ear have been obtained. Fletcher finds for a frequency of 2000 cycles that the minimum audible sound corresponds to a pressure of 6×10^{-4} dynes per sq. cm. and to an amplitude of 2×10^{-9} cm. and that the ear still recognizes as sound the effect of pressures up to 6,000 dynes per sq. cm. and of amplitudes up to 1.7×10^{-4} cm. "The total audible range of intensity can be traversed in about 270 barely perceptible steps."

The problem of making the voice of a speaker audible to a large audience is a complicated one. The auditorium furnishes echoes, foci, reverberation and resonance. Waves from the loud speaker acting on the microphone along with the original voice may produce singing or distortion; indeed distortion is likely to be produced by each step in the chain of acoustic and electrical processes employed. "Fortunately the ear is so bad a judge of intensity of sound that exact equality in the magnification of the frequencies in a complex sound is not required. In fact certain frequencies of the speech range may be magnified in intensity ten times as much as certain other frequencies in that range, without the ear noticing the inequality of amplification; and certain frequencies may be altogether omitted by the apparatus from the reproduction of a speaker's voice without loss of intelligibility or naturalness." When an address is to be made audible to a huge audience it is better to

distribute the magnified electric current among several loud speakers rather than to concentrate it upon a single one. When all the component frequencies of a complex sound are magnified in the same ratio so that there is no actual distortion the listener hears the sound as badly distorted. It is thus seen why investigations in the psychology of hearing have so great a practical importance.

G. F. S.

Measurement of the Absorbing Power of Materials for Sound by the Stationary Wave Method. A. H. DAVIS AND E. J. EVANS. (*Proc. Roy. Soc.*, A, 804.) As more and more attention is given to the acoustical properties of auditoriums and to methods of remedying their defects it becomes increasingly necessary to have convenient and rapid methods of measuring the absorptive power of materials. Much of the work hitherto done has been carried out by disposing the material in a room and measuring its effect on the length of time that a standard sound reverberates. This is a trial under practical conditions, but large samples of the material are needed and introducing cushions into a chamber, as Sabin did, was a long process. "The stationary wave method of determining the absorption coefficient of a material employs plane waves of sound at perpendicular incidence. It requires the use of only small samples of material and provides a rapid and convenient means of obtaining useful information. A long pipe is provided with a source of sound at one end and is closed at the other by the test specimen. Sound waves from the source travel down the pipe and are reflected by the specimen to an extent depending on its absorbing power. The superposition of the incident and reflected waves gives rise to a stationary wave system, and the pressure amplitude varies continuously along the pipe, going through a series of maximum and minimum values. The same description applies to the velocity amplitude, with the difference that the pressure maxima coincide in position with the velocity minima and *vice versa*." From the ratio which either minimum amplitude bears to the corresponding maximum it is possible to calculate the absorption coefficient of the material. The ear, a telephone receiver, a Rayleigh disc have all been used in getting this ratio. The authors in their measurements used an exploring tube inserted within the pipe and connected to a loud-speaker movement with moving coil. The E.M.F. at the terminals of the coil was in proportion to the pressure on the diaphragm of the exploring tube.

The absorption coefficient was found to depend on whether the material investigated was fastened rigidly to the steel backing plate. The difference was little in such soft materials as felt but con-

siderable in the case of board-like substances. For the sake of uniformity all were rigidly supported, unless otherwise stated. In agreement with Paris a change of phase upon reflection was found to occur. This caused a shift in the stationary wave system of about 1 cm.

"In many cases, for certain regions of thickness, an increase of thickness produced a decrease of absorption coefficient. This effect, which does not appear to have been noted before, can be explained on theoretical grounds by taking account of the interference between sound reflected directly from the front surface and that emerging from the material after reflection at the backing surface. The phenomenon is analogous to the reflection of light by plane parallel surfaces in optics." In the case of cotton waste the coefficient is as follows for different thicknesses,—

Thickness in cm.	1	2	3	4	5	7	10	15	20
Coefficient	.2	.4	.7	.96	.98	.9	.93	.94	.97

When holes were made in a board-like material the coefficient of absorption was greatly increased, becoming from 5 to 7 times as great. With all materials the absorption was greater with shorter than with longer wave-lengths. The results are given for several commercial absorbents as well as for felt etc. The steel backing plate scarcely absorbed at all, its coefficient being from .002 to .01.

The results obtained by this method are by no means the same as those derived by the study of reverberation. The ratio between the reverberation coefficient and the stationary wave coefficient varies according to material from 1 to 4.8. This divergence may be due to the different ways of mounting the test specimens in the two methods.

G. F. S.

THE FRANKLIN INSTITUTE

OFFICERS FOR 1930

President	NATHAN HAYWARD
Vice-presidents	HENRY HOWSON C. C. TUTWILER WALTON FORSTALL W. CHATTIN WETHERILL
Secretary	HOWARD MCCLENAHAN
Assistant Secretary and Librarian	ALFRED RIGLING
Treasurer	BENJAMIN FRANKLIN
Assistant Treasurer	MARSHALL S. MORGAN
Controller	WILLIAM F. JACKSON, JR.

Board of Managers

HENRY B. ALLEN
GEORGE H. BENZON, JR.
EDWARD G. BUDD
FRANCIS T. CHAMBERS
G. H. CLAMER
WALTON CLARK
CYRUS H. K. CURTIS
CHARLES DAY
SAMUEL S. FELS
ARTHUR W. GOODSPEED
CLARENCE A. HALL
GEORGE A. HOADLEY
A. ATWATER KENT
L. H. KINNARD
ROBERT W. LESLEY
MALCOLM LLOYD, JR.
ROBERT MCLEAN
CHARLES PENROSE
JAMES S. ROGERS
GEORGE D. ROSENGARTEN
E. H. SANBORN
HASELTINE SMITH
SAMUEL T. WAGNER